

# **Guidelines for Mathematics Laboratory in Schools**

## **Class IX**

Central Board of Secondary Education  
Preet Vihar, Delhi – 110092.

---

# 1. Introduction

## 1.1 Rationale

Mathematics is erroneously regarded as a difficult subject to understand, meant only for persons of 'higher' mental ability. It arouses fear among any students, which in turn creates resistance to learning and results in an adverse effect on their attainment. But actually, school mathematics is within the reach of any average student. What is needed is to create the right ambience of learning mathematics in every school.

Mathematics needs to be learnt with a sense of joy and delight. It needs to be related, where possible, to life-oriented activities, to create interest in the subject. Mathematical faculty and intuition develop not only through theory and problems given in mathematics textbooks but also through a variety of activities involving concrete objects. Activities can be engaging as well as instructive.

With this in mind, CBSE has endeavoured to introduce the idea of mathematics laboratory in schools.

Some of the ways in which activities in a mathematics laboratory could contribute to learning of the subject are:

- It provides an opportunity to students to understand and internalise the basic mathematical concepts through concrete situations. It lays down a sound base for more abstract thinking.
- The laboratory gives greater scope for individual participation. It encourages students to become autonomous learners and allows an individual student to learn at his or her own pace.
- It helps build interest and confidence among the students in learning the subject.
- It provides opportunity to students to repeat an activity several times. They can revisit and rethink a problem and its solution. This helps them develop metacognitive abilities.
- It allows and encourages students to discuss, think and assimilate the concepts in a better manner through group learning.
- It provides opportunity to students to understand and appreciate the applications of mathematics in their surroundings and real life situations.
- It widens the experimental base and prepares the ground for better learning of new areas in the subject.
- An activity involves both the mind and hands of the student working together, which facilitates cognition.

## 1.2 National Curriculum Framework and Board's Initiatives.

The National Curriculum Framework for school education (NCFSE) developed by NCERT emphasizes that mathematics learning should be facilitated through activities from the very beginning of school education. These activities may involve the use of concrete materials, models, patterns, charts, pictures, posters, games, puzzles and experiments. The Framework strongly recommends setting up of a mathematics

laboratory in every school in order to help exploration of mathematical facts through activities and experimentation.

With the objective of meeting these national requirements, aspirations and expectations, the Central Board of Secondary Education immediately issued directions to its affiliated schools to take necessary action in this regard. Simultaneously, a document on '*Mathematics Laboratory in schools – towards joyful learning*' was brought out by the Board and made available to all the schools. This document primarily aimed at sensitizing the schools and teachers to the philosophy of a mathematics laboratory, creating awareness among schools as to how mathematics laboratory will help in improving teaching and learning of the subject and providing general guidelines to school on setting up and using a mathematics laboratory. Besides, it also included a number of suggested hands-on activities related to concepts in mathematics for Class III to Class X. Teachers were advised to design more activities of similar nature to suit the requirements of the classes and students under their charge.

There has been a very encouraging response to this initiative from the schools and a large number of them have already established reasonably functioning mathematics laboratories. However, the Board has been receiving queries and observations from many quarters with the request to provide more detailed guidelines to set up such a laboratory, particularly with regard to its size and design, physical infrastructure, materials required and human resources. In addition to including specific activities and project work for Class IX, the present document aims at clarifying these various matters.

### **1.3. About the present document**

The present document has three clear objectives. Firstly, it aims at providing detailed guidelines to schools with regard to the general layout, physical infrastructure, materials and human resources for a mathematics laboratory. This would, it is expected, clear doubts about the minimum requirements for setting up of such a laboratory. Secondly, it includes details of all Class IX syllabus related activities to be done by the students during the academic year. Thirdly, it gives a few specific examples of projects. This is intended to help the schools to have an idea of the nature of project work to be undertaken by the students. Since the schools have already been given directions in relation to setting up of a mathematics laboratory by 31<sup>st</sup> March, 2005 through circular No.....dated....., it is expected that necessary initiatives have been taken and the desired facilities are available in schools. The schools are now expected to extend and expand these facilities to carry out Class IX syllabus activities from the academic session starting April 2005. Another circular No.....dated.....has also been issued in relation to the introduction of 20% internal assessment scheme in the subject in Class IX from the ensuing academic session beginning April 2005. The said circular clarifies that the internal assessment is to be given on the basis of performance of an individual in the practical work. The details of assessment in practical work are given in the later sections of this document.

## 2. Mathematics Laboratory

### 2.1 What is a Mathematics Laboratory ?

Mathematics laboratory is a room wherein we find collection of different kinds of materials and teaching/learning aids, needed to help the students understand the concepts through relevant, meaningful and concrete activities. These activities may be carried out by the teacher or the students to explore the world of mathematics, to learn, to discover and to develop an interest in the subject.

### 2.2 Design and general layout.

A suggested design and general layout of laboratory which can accommodate about 30 students at a time is given on page.....The design is only a suggestion. The schools may change the design and general layout to suit their own requirements.

### 2.3 Physical Infrastructure and Materials

It is envisaged that every school will have a Mathematics Laboratory with a general design and layout as indicated on page.....with suitable change, if desired, to meet its own requirements. The minimum materials required to be kept in the laboratory may include all essential equipment, raw materials and other essential things to carry out the activities included in the document effectively. The quantity of different materials may vary from one school to another depending upon the size of the group. Some of the essential materials required are given on page 11.

### 2.4 Human Resources

It is desirable that a person with minimum qualification of graduation (with mathematics as one of the subjects) and professional qualification of Bachelor in Education be made incharge of the Mathematics Laboratory. He/she is expected to have special skills and interest to carry out practical work in the subject. It will be an additional advantage if the incharge possesses related experience in the profession. The concerned mathematics teacher will accompany the class to the laboratory and the two will jointly conduct the desired activities. A laboratory attendant or laboratory assistant with suitable qualification and desired knowledge in the subject can be an added advantage.

### 2.5 Time Allocation for activities.

It is desirable that about 15% - 20% of the total available time for mathematics be devoted to activities. Proper allocation of periods for laboratory activities may be made in the time table.

## Scheme of Evaluation

As an extension of the Board's intention to make learning of mathematics a more meaningful exercise, it has been decided to introduce the scheme of internal assessment in the subject. The objective is not merely to evaluate the learner in a public examination and award marks but to promote and encourage continuous

self-actualised learning in the classroom and in the extended hours of schooling. This internal assessment will have a weightage of 20 marks as per the following break up :

Year-end Evaluation of activities	:	10 marks
Evaluation of project work	:	05 marks
Continuous assessment	:	05 marks

The year-end assessment of practical skills will be done during an organized session of an hour and a half in small groups as per the admission ..... convenience of the schools with intimation to the Board. The break up of 10 marks could be as under :

Complete statement of the objective of activity	:	1 mark
Design or approach to the activity	:	2 marks
Actual conduct of the activity	:	3 marks
Description /explanation of the procedure followed	:	2 marks
Result and conclusion	:	2 marks

Out of all the activities given in the document, every student may be asked to complete a minimum of 15 marked activities during the academic year and be examined in one of these activities. He/she should be asked to maintain a proper activity record for this work done during the year.

The schools would keep a record of the conduct of this examination for verification by the Board, whenever necessary, for a period of six months. This assessment will be internal and done preferably by a team of two teachers.

## Evaluation of project work

Every student will be asked to do one project based on the concepts learnt in the classroom but as an extension of learning to real life situations. This project work should not be repetition or extension of laboratory activities but should infuse new elements and could be open ended and carried out beyond the school working hours.

Five marks weightage could be further split up as under :

Identification and statement of the project:	01 mark
Design of the project	01 mark
Procedure /processes adopted	02 marks
Interpretation of results	01 mark

## Continuous Assessment

Continuous assessment could be awarded on the basis of performance of students in their first and second terminal examinations. The strategy given below may be used for awarding internal assessment in Class IX :

- (a) Reduce the marks of the first terminal examination to be out of ten.
- (b) Reduce the marks of the second terminal examination to be out of ten.
- (c) Add the marks of (a) and (b) above and get the achievement of the learner out of twenty marks.
- (d) Reduce the total in (c) above to the achievement out of five marks.
- (e) This score may be added to score of year-end evaluation of activities and to score in project work to get the total score out of 20 marks.

It is expected that the marks obtained by a student in theory examination (80) and laboratory work (20) be indicated separately in the achievement card.

# List of activities

**1A.** To carry out the following paper folding activities:

Finding –

1. the mid point of a line segment,
2. the perpendicular bisector of a line segment,
3. the bisector of an angle,
4. the perpendicular to a line from a point given outside it,
5. the perpendicular to a line at a point given on the line,
6. the median of a triangle.

**1B.** To carry out the following activities using a geoboard:

1. Find the area of any triangle.
2. Find the area of any polygon by completing the rectangles.
3. Obtain a square on a given line segment.
4. Given an area, obtain different polygons of the same area.

**2.** To obtain a parallelogram by paper–folding.

**3.** To show that the area of a parallelogram is product of its base and height, using paper cutting and pasting. (Ordinary parallelogram and slanted parallelogram)

**4.** To show that the area of a triangle is half the product of its base and height using paper cutting and pasting. (Acute, right and obtuse angled triangles)

**5.** To show that the area of a rhombus is half the product of its diagonals using paper cutting and pasting.

**6.** To show that the area of a trapezium is equal to half the product of its altitude and the sum of its parallel sides and its height, using paper cutting and pasting.

**7.** To verify the mid point theorem for a triangle, using paper cutting and pasting.

**8.** To divide a given strip of paper into a specified number of equal parts using a ruled graph paper.

**9.** To illustrate that the perpendicular bisectors of the sides of a triangle concur at a point (called the circumcentre) and that it falls

- a. inside for an acute-angled triangle.
- b. on the hypotenuse of a right-angled triangle.
- c. outside for an obtuse-angled triangle.

10. To illustrate that the internal bisectors of angles of a triangle concur at a point (called the incentre), which always lies inside the triangle.
11. To illustrate that the altitudes of a triangle concur at a point (called the orthocentre) and that it falls
  - a. inside for an acute angled triangle.
  - b. at the right angle vertex for a right angled triangle.
  - c. outside for an obtuse angled triangle.
12. To illustrate that the medians of a triangle concur at a point (called the centroid), which always lies inside the triangle.
- 13A. To give a suggestive demonstration of the formula that the area of a circle is half the product of its circumference and radius. (Using formula for the area of triangle)
- 13B. To give a suggestive demonstration of the formula that the area of a circle is half the product of its circumference and radius. (Using formula for the area of rectangle)
14.
  - 1) To verify that sum of any two sides of a triangle is always greater than the third side.
  - 2) To verify that the difference of any two sides of a triangle is always less than the third side.
15. To explore criteria of congruency of triangles using a set of triangle cut outs.
16. To explore the similarities and differences in the properties with respect to diagonals of the following quadrilaterals – a parallelogram, a square, a rectangle and a rhombus.
17. To explore the similarities and differences in the properties with respect to diagonals of the following quadrilaterals – a parallelogram, a square, a rectangle and a rhombus.
18. To show that the figure obtained by joining the mid points of the consecutive sides of any quadrilateral is a parallelogram.
19. To make nets for a right triangular prism and a right triangular pyramid (regular tetrahedron) and obtain the formula for the total surface area.
20. To verify Euler's formula for different polyhedra: prism, pyramids and octahedron.

21. Obtain length segments corresponding to square roots of natural numbers using graduated wooden sticks.
22. To verify the identity  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ , for simple cases using a set of unit cubes.
23. To verify the identity  $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ , for simple cases using a set of unit cubes.
24. To verify the identity  $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ , for simple cases using a set of unit cubes.
25. To verify the identity  $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$ , for simple cases using a set of unit cubes.
26. To interpret geometrically the factors of a quadratic expression of the type  $x^2 + bx + c$ , using square grids, strips and paper slips.
27. To obtain mirror images of figures with respect to a given line on a graph paper.

## Group Activities

1. To find the percentage of students in a group of students who write faster with their left hand / right hand.
2. To help the students establish interesting mathematical relationships by measuring some parts of the body.

## List of projects given as examples in the booklet

- 1. Observing interesting patterns in cricket match.**  
Comparison of the performance of two teams in a one-day international cricket match.
- 2. Design a crossword puzzle with mathematical terms**  
To review mathematics vocabulary, to give the opportunity for creative expressions in designing puzzles, to act as a means of monitoring the study of a given unit and to give recreation.
- 3. A measuring task**  
To investigate your local athletics track to see whether it is marked fairly for runners who start on different lines.

#### 4. Project in history of mathematics

- i. Study various aspects of Pythagoras theorem.
- ii. Investigation of various historical aspects of number  $\pi$ .

## Suggested list Of Projects

### P1 Cricket

Collect data on runs scored in each over for a one-day international (ODI) cricket match and obtain frequency distribution between runs and overs. Do this for both the teams and also for the first 25 and the remaining overs of the match. Observe any interesting features of the match. Compare it with similar analysis for a few other ODI's.

### P2 Age profile in your neighbourhood

Survey any 30 households in your locality and collect data on the age of the persons. Determine the age profile (number of persons Vs age) for men and women. Report any significant observation from the data.

### P3 Educational Background in your neighbourhood

Survey any 30 households in your locality and collect data on the educational background of the persons. Obtain significant observations from your data.

### P4 Number of Children in a family in your neighbourhood

Survey any 50 households in your locality and collect data on the number of children (male and female) in each family. Report any significant observation.

### P5 Making of Platonic solids

Obtain and construct the nets of five platonic solids. Make these solids and observe the properties (number of faces, edges and vertices) of the solids. Try to find out, why there are only five platonic solid. (Try taking regular hexagon)

### P6 History of Mathematics

Refer history of mathematics sources from your library or Internet and prepare a poster or a document on any topic of your interest. The students can choose several topics from history of mathematics, for doing a project. For instance the topic can be about an Indian mathematician or the concept of zero in various ancient civilizations.

### P7 Mathematics line designs

Using strings obtain interesting designs and patterns. Use threads and shapes made by cardboard, try to make designs on it by making slits on the cardboard. Observe different patterns on it.

### P8 Computer project

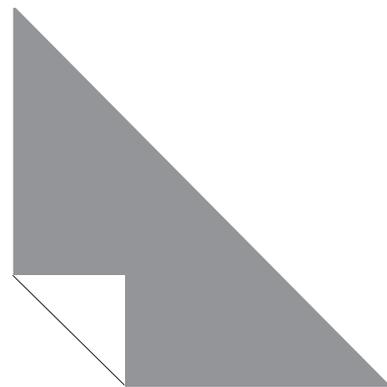
Using a spreadsheet programme on a PC obtain the graph of the equation  $ax^2 + bx + c = 0$  for a different values of a, b and c and note the interesting features and patterns. Interested students can also try for quadratic equations.

## List of methods and materials used in the mathematics laboratory

- i. Paper folding
- ii. Collage (Paper cutting & pasting)
- iii. Unit Cubes (wooden or any material)
- iv. Geo-board, rubber band
- v. Transparency sheets, cello tape
- vi. Graph paper
- vii. Pins & threads
- viii. Broom sticks
- ix. Chart papers, glazed papers, sketch pens.
- x. Stationery

## Activity 1A

### Basic paper folding activity



#### Objectives

To carry out the following paper folding activities:

Finding -

1. the mid point of a line segment,
2. the perpendicular bisector of a line segment,
3. the bisector of an angle,
4. the perpendicular to a line from a point given outside it,
5. the perpendicular to a line at a point given on the line,
6. the median of a triangle.

#### Pre-requisite knowledge

Meaning of the basic geometrical terms such as perpendicular bisector, angle bisector and median.

#### Materials required

Rectangular sheets of coloured paper, a pair of scissors.

#### Procedure

1. Make a line segment on the paper, by folding the paper in any way. Call it AB. Fold the line segment AB in such a way that A falls on B, halving the length of AB. Mark the point of intersection of line segment AB and the crease formed by folding the paper. This gives the mid-point E of segment AB. [Fig 1A (a)]
2. Fold AB in such a way that A falls on B, thereby creating a crease EF. This crease is the perpendicular bisector of AB. [Fig 1A (b)]
3. Cut a triangle from a coloured paper and name it PQR. Fold along the vertex P of the triangle in such a way that the sides PQ and PR coincide with each other. The crease PF formed is the angle bisector of the angle P. [Fig 1A (c)]
4. Draw a line segment AB and take a point P outside it. Move B along BA till the fold passes through P and crease it along that line. The crease formed is the perpendicular to AB from point P. [Fig 1A (d)]
5. Draw a line AB and take a point C on it. Move B along the line BA till the fold passes through C and crease it at along that line. The crease so formed is the perpendicular to AB at the point C on it. [Fig 1A (e)]
6. Cut out a triangle ABC. Find the mid-points of the sides by the method given in step 1. Join A, B, C to the respective mid-points of opposite sides, BC, CA and AB by paper folding. The creases formed are the medians of the triangle. [Fig 1A (f)]

#### Observations

In some cases the students may like to verify the results obtained in this activity by actual measurement.

### Learning Outcomes

Students are exposed to the basic features of paper folding. They will appreciate that several geometrical constructions can be carried out very simply by paper folding.

### Remark

The teacher should ensure that students get enough practice in this activity, since this is basic for many of the subsequent laboratory activities given in the booklet.

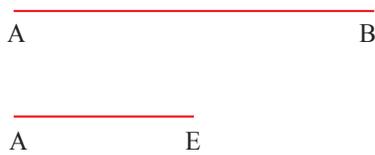


Fig 1A (a)

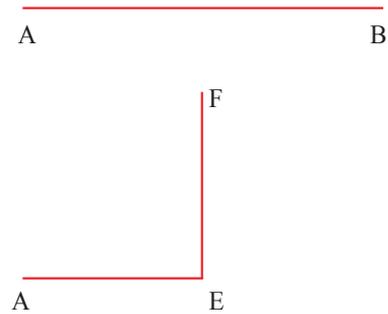


Fig 1A (b)

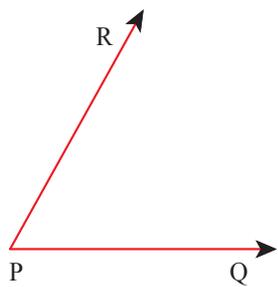


Fig 1A (c)

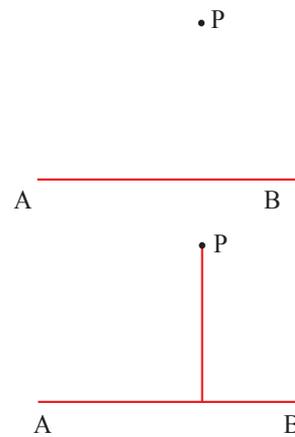
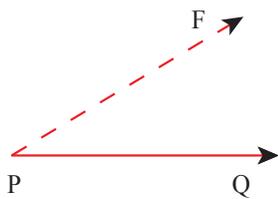


Fig 1A (d)

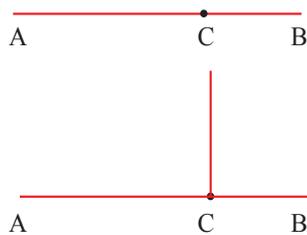


Fig 1A (e)

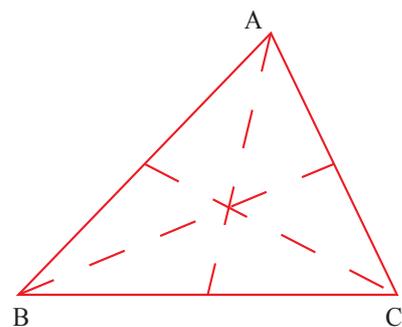
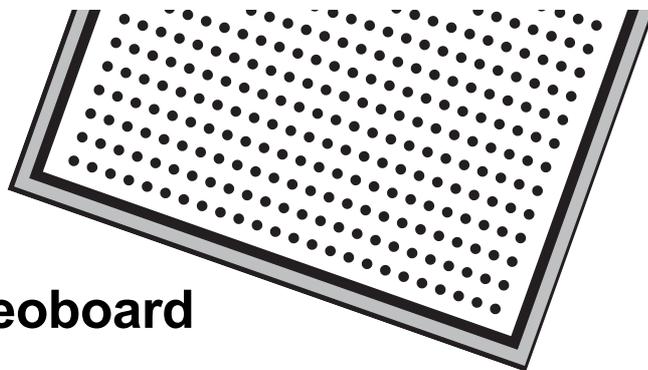


Fig 1A (f)

## Activity 1B



### Basic activities using Geoboard

#### Objectives

To carry out the following activities using a geoboard:

1. Find the area of any triangle.
2. Find the area of any polygon by completing the rectangles.
3. Obtain a square on a given line segment.
4. Given an area, obtain different polygons of the same area.

#### Pre-requisite knowledge

Basic understanding about names and properties of the shapes.

#### Materials required

Square wooden board of 10" × 10", nails with small heads, hammer, rubber bands, marking pen.

#### Procedure

##### *Making Geoboard*

Take a wooden square base. Mark an array of 10 × 10 dots on this wooden base. Fix nails on these equidistant dots.

##### *Activities on Geoboard*

1. Practice making various geometric figures using rubber bands on the geoboard.
2. Find area of regular quadrilaterals by counting number of unit squares in it.
3. Find the area of triangles by completing triangles into rectangles and halving them. Verify the result using formula for area of triangles.
4. Make any irregular polygon, complete the rectangles at the non-horizontal/vertical sides of the polygon, and find the area of various polygons. [Fig 1B (a)]
5. Given any line segment on the geoboard, ask students to form a square, with this line segment as the base.
6. If the area of any polygon is given, find the different possibilities of shapes with same area. In Fig 1B b) the area of all the shapes is same and is 12 sq.units.

#### Observations

1. Students observe that the area of any shape is equal to the number of unit squares in the space occupied by that shape.
2. Students find it interesting to find the area of any irregular polygon by the method of completing rectangles.

#### Learning outcomes

1. Students learn the concept of area as number of unit squares in the space occupied by the shape.

2. They learn the relation between area and the formula for the area of any shape.
3. The activities like constructing a square on a given line and making various polygons of a given area, enable students to think logically. It also helps them to develop spatial understanding about the objects.

**Remark**

Many theorems can be verified on the geoboard, for e.g. the area of any triangle on the same base and between the same parallel lines is half that of the parallelogram.

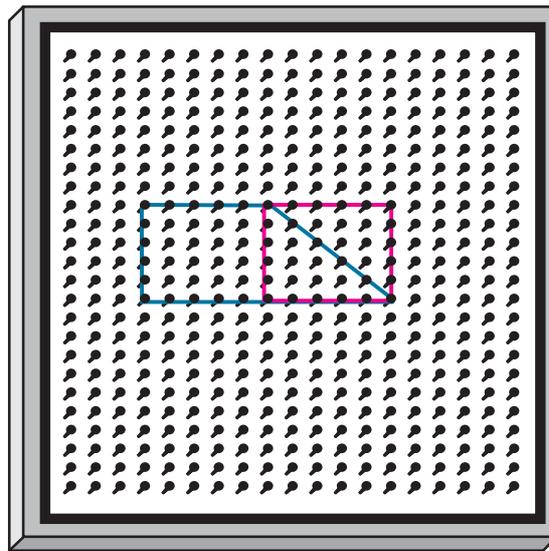


Fig 1B (a)

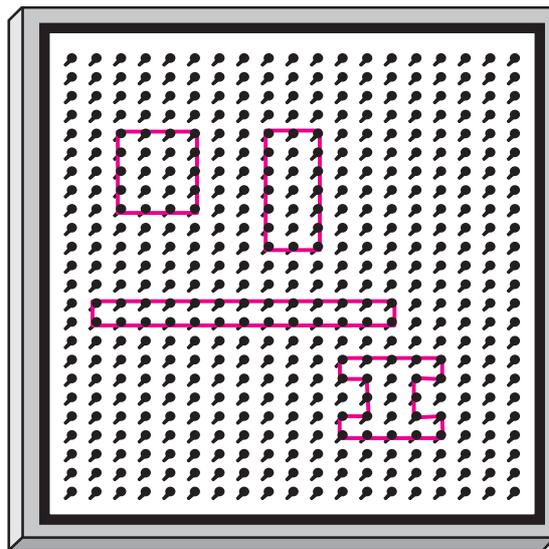
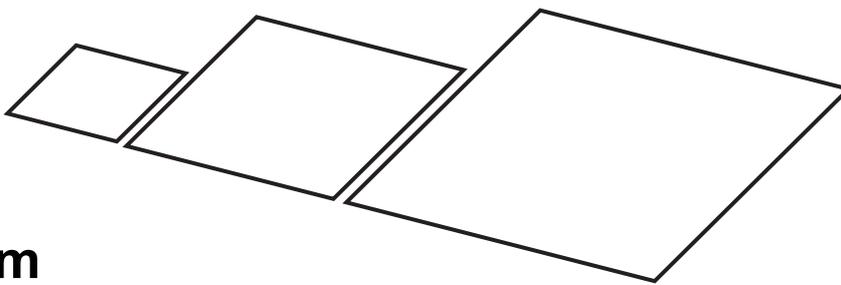


Fig 1B (b)

## Activity 2



# Parallelogram

### Objective

To obtain a parallelogram by paper folding.

### Pre-requisite knowledge

1. Familiarity with activity 1A.
2. To know that, a parallelogram is a quadrilateral in which the pair of opposite sides are parallel.

### Material required

Rectangular sheet of paper.

### Procedure

1. Take a rectangular sheet of paper.
2. Fold it parallel to its breadth at a convenient distance and make a crease (1).
3. Obtain a crease perpendicular to the crease (1) at any point on it and call it crease (2).
4. Obtain a third crease perpendicular to crease (2) at any point on crease (2) and call it as crease (3).
5. Mark crease (1) and (3) with pencil. This represents a pair of parallel lines.
6. Make a fold, cutting the creases (1) and (3), call it crease (4). Adopting the method used for getting a pair of parallel lines as explained in steps 1 to 5, get a fold parallel to crease (4), call this as crease (5). [Fig 2 (a)]

### Observations

1. Crease (1) and (3) are parallel.
2. Crease (4) and (5) are parallel.
3. The enclosed figure is parallelogram.

### Learning outcomes

1. The students learn to make a line parallel to a given line and a parallelogram by paper folding.
2. They internalise the simple properties of a parallelogram.

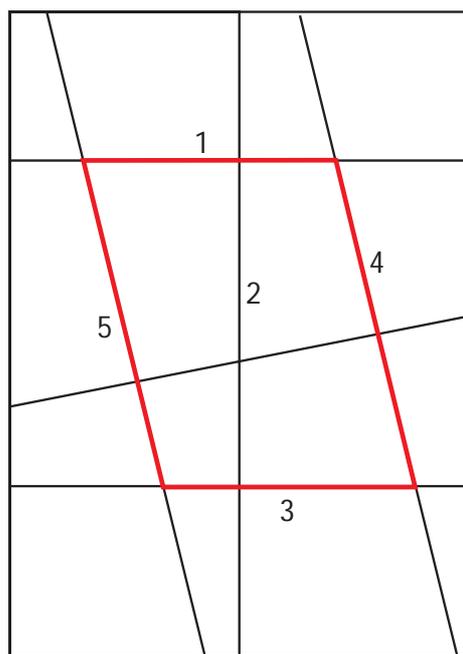
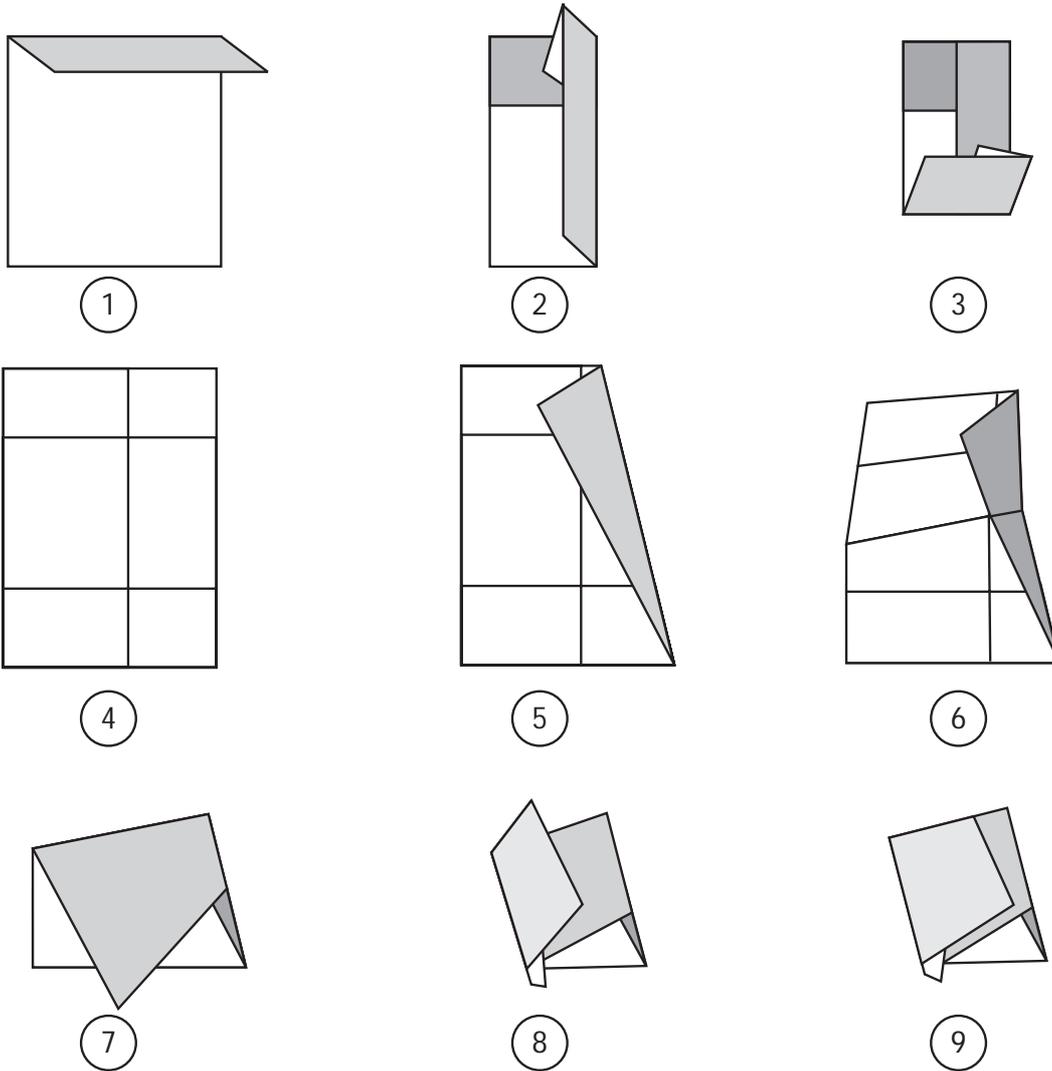


Fig 2 (a)

## Activity 3

# Area of a parallelogram

### Objective

To show that the area of a parallelogram is product of its base and height using paper cutting and pasting.

### Pre-requisite knowledge

1. Familiarity with Activity 1A and 2.
2. Formula for the area of a rectangle

### Material Required

Glazed paper, pencil, a pair of scissors, gum.

### Procedure

1. Make a parallelogram by paper folding. Call it ABCD.
2. Cut out the parallelogram with the help of a pair of scissors.
3. Obtain a perpendicular from D to AB meeting AB at E. [Fig 3 (a)]
4. Cut and remove the triangle AED and align AD with BC. Call the displaced segment AE as AE'. [Fig 3 (b)]
5. Verify using a scale that EBE' are collinear.
6. Verify that CE' is perpendicular to EBE' and  $EE' = CD$
7. Observe that the figure obtained is a rectangle. [Fig 3 (b)]

### Observations

1. Area of parallelogram ABCD = area of rectangle EE'CD = (length  $\times$  breadth)  
 $= EE' \times CE'$ .
2. Area of parallelogram = base  $\times$  height.

### Learning Outcomes

1. The students will be able to infer that the area of parallelograms with the same base and the same height are identical. This leads to the well-known theorem: "Parallelograms between the same parallel lines and with the same base are equal in area."
2. The students will understand geometrically how the formula for the area of parallelogram (base  $\times$  height) works.

### Remarks

1. Teacher can ask the students to draw perpendiculars from different vertices on opposite sides and verify that the area of parallelogram is product of base and height, independent of which side is taken as the base.

2. Teacher can discuss various cases of parallelograms for verification of the formula where the perpendicular falls outside the base. [Fig 3 (c)]

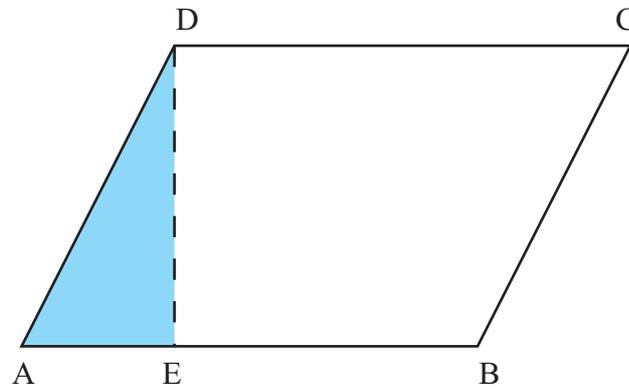


Fig 3 (a)

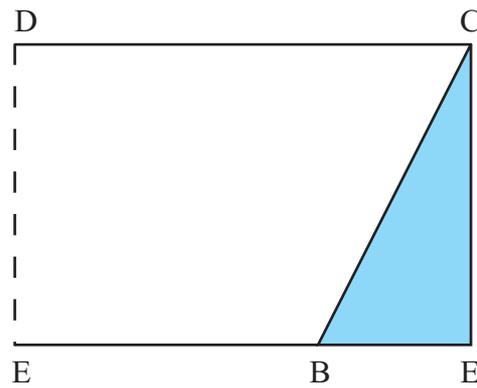


Fig 3 (b)

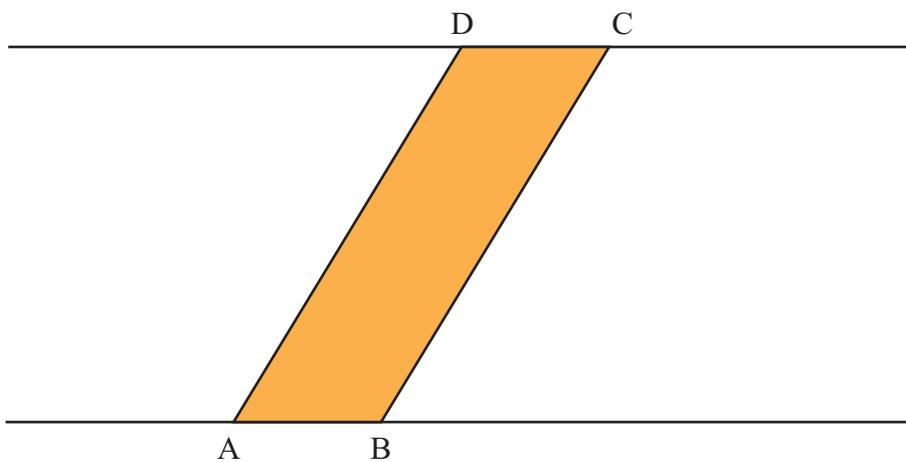
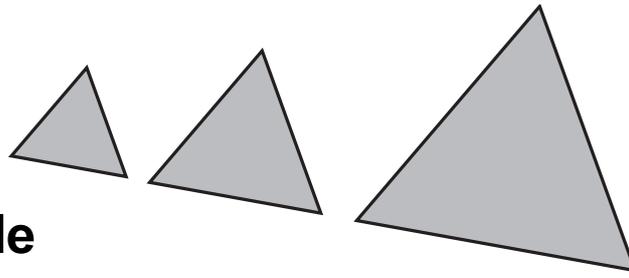


Fig 3 (c)

## Activity 4



# Area of triangle

### Objective

To show that the area of a triangle is half the product of the base and the height using paper cutting and pasting.

### Pre-requisite knowledge

1. Familiarity with activity 1A.
2. Formula for the area of a rectangle.
3. A diagonal of a parallelogram divides it into two congruent triangles.

### Material Required

Chart paper, pencil, compass, scale, a pair of scissors, cello tape.

### Procedure

For Right angle triangle

1. Cut a right angle triangle. [Fig 4 (a)]
2. Cut a triangle congruent to the right angle triangle.
3. Align the hypotenuse of the two triangles to obtain a rectangle. [Fig 4 (b)]

### Observations

The students observe that two congruent triangles aligned on hypotenuse forms a rectangle.

They can see that area of rectangle = area of two congruent triangles.

Area of rectangle = base  $\times$  height.

Therefore, Area of triangle =  $\frac{1}{2} \times$  Area of rectangle =  $\frac{1}{2} \times$  base  $\times$  height.

### Procedure

For Acute Angle Triangle

1. Cut an acute angle triangle and draw the perpendicular from the vertex to the opposite side. [Fig 4 (c)]
2. Cut a triangle congruent to it and cut this triangle along the perpendicular. [Fig 4 (d)]
3. Align the hypotenuse of these cut outs to the given triangle in order to obtain a rectangle. [Fig 4 (e)]

### Observations

The students observe that two congruent triangles aligned in a specific way forms a rectangle.

They can see that area of rectangle = area of two congruent triangles.

Area of rectangle = base  $\times$  height.

Therefore, Area of triangle =  $\frac{1}{2} \times$  Area of rectangle =  $\frac{1}{2} \times$  base  $\times$  height.

## Procedure

For Obtuse Angle Triangle

1. Cut an obtuse angle triangle. [Fig 4 (f)]
2. Cut a triangle congruent to this obtuse angle triangle.
3. Align the greatest side of the two triangles in order to obtain parallelogram. [Fig 4 (g)]

## Observations

The students observe that aligning these two congruent triangles forms a parallelogram.

They can see that area of the parallelogram = area of two congruent triangles.

Area of parallelogram = base  $\times$  height.

Area of triangle =  $\frac{1}{2} \times$  area of parallelogram =  $\frac{1}{2} \times$  base  $\times$  height.

## Learning Outcome

The students may infer that area of each triangle is half the product of its base and height irrespective of the sides and angles of triangle.

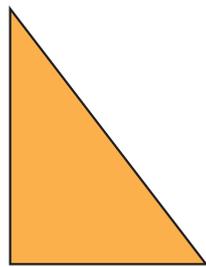


Fig 4 (a)

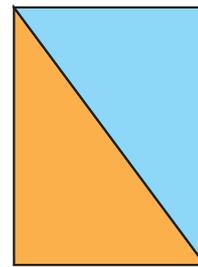


Fig 4 (b)

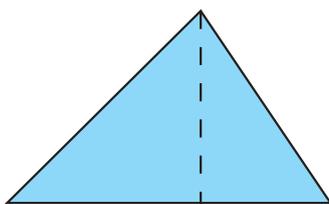


Fig 4 (c)

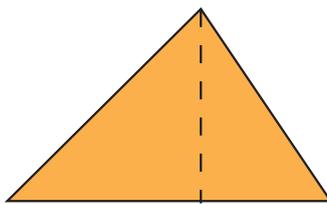


Fig 4 (d)

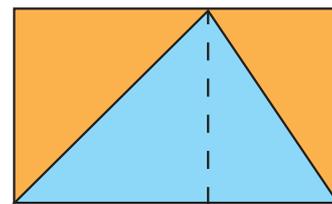


Fig 4 (e)

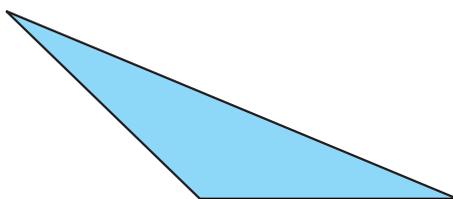


Fig 4 (f)

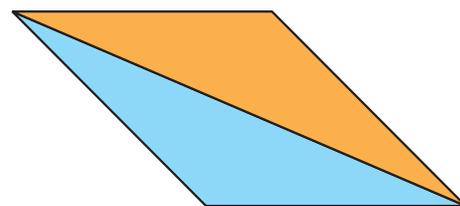
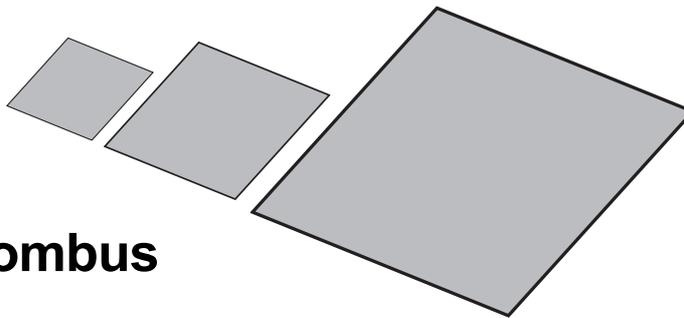


Fig 4 (g)

## Activity 5



# Area of a rhombus

### Objective

To show that the area of a rhombus is half the product of its diagonals using paper cutting and pasting.

### Pre-requisite knowledge

1. Properties of a rectangle and a rhombus.
2. Formula for area of a triangle and a rectangle.
3. Concept of congruency.

### Material Required

Colored papers, sketch pens, geometry box, a pair of scissors, fevicol and eraser.

### Procedure

1. Draw a rectangle ABCD with length  $d_2$  and breadth  $d_1$  units on a coloured paper.
2. Mark points E, F, G and H as mid points of the sides AD, DC, CB and BA respectively of sides of the rectangle ABCD drawn in step 1. [Fig 5 (a)]
3. Join HF and EG. Mark their intersection as point O. Fold the rectangle ABCD along EG and HF dividing the rectangle ABCD into four congruent rectangles, namely OEAH, OEDF, OFCG and OGBH.
4. Divide each of the four rectangles into two congruent triangles by drawing their respective diagonals. [Fig 5 (a)]

### Observations

1. As the smaller rectangles are congruent, their diagonals EH, HG, GF, FE are equal. Thus EHG is a rhombus.
2. In the rectangle AHOE, triangles AHE and EHO are congruent and hence equal in area.
3. Thus area of the right triangle EOH is half the area of the rectangle AEOH and similarly, the area of right triangles HOG, GOF, FOE are half the area of the rectangles HBGO, OGCF and FOED respectively.
4. Thus the area of rhombus =  $\frac{1}{2} \times$  area of rectangle

$$= \frac{1}{2} \times d_1 \times d_2$$

$$= \frac{1}{2} \times \text{product of diagonals}$$

### Learning Outcomes

1. This activity shows construction of the rhombus by paper folding.
2. Students understand geometrically, that the area of rhombus is half the product of its diagonals.

**Remark**

This activity may be extended to the case of a Kite and the same formula may be verified.

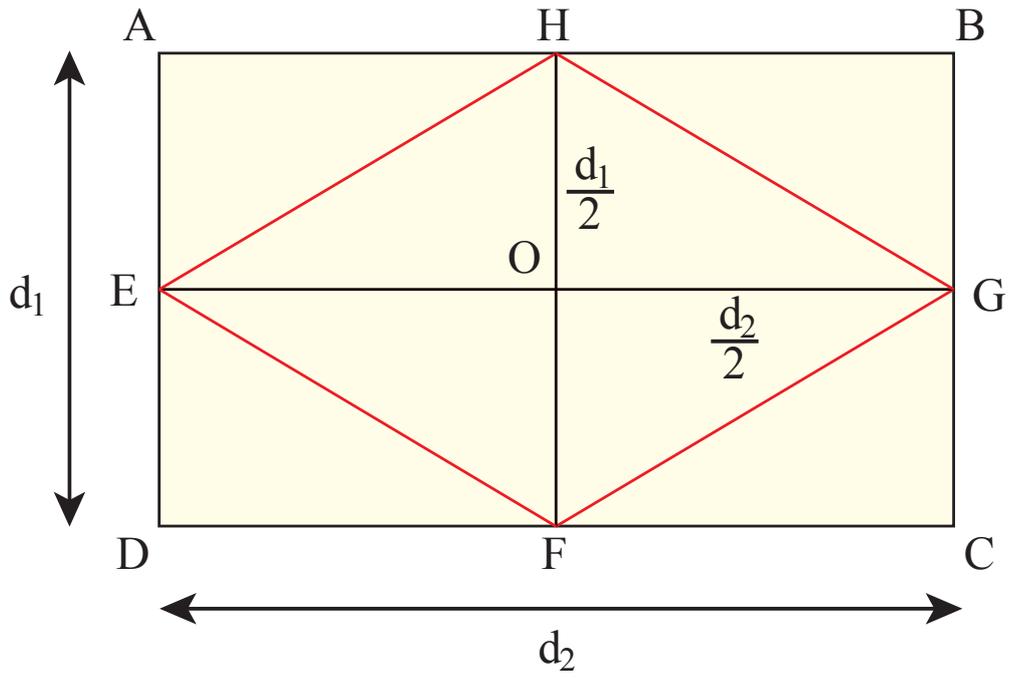
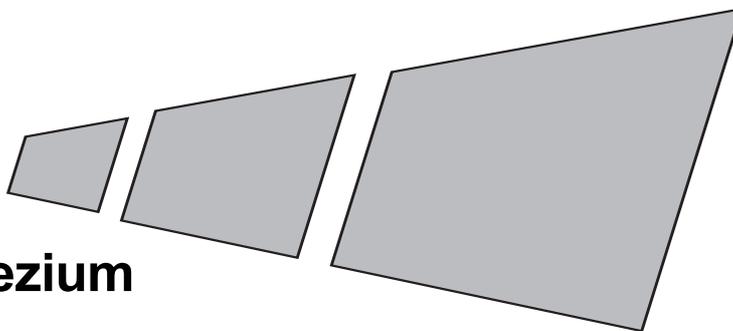


Fig 5 (a)

## Activity 6



### Area of a trapezium

#### Objective

To show that the area of a trapezium is equal to half the product of its altitude and the sum of its parallel sides, using paper cutting and pasting.

#### Pre-requisite knowledge

1. A trapezium is a quadrilateral with one pair of opposite sides parallel.
2. A quadrilateral is a parallelogram if a pair of its opposite sides are parallel and equal to each other.

#### Material Required

Coloured paper, a pair of scissors, gum.

#### Procedure

1. Take two sheets of coloured paper.
2. Cut two identical trapeziums ABCD and PQRS. [Fig 6 (a)]
3. Paste them together as shown in Fig 6(b) to obtain a quadrilateral RBCQ.

#### Observations

The two trapezia add up to form a parallelogram whose base RB is the sum of the two parallel sides of the trapezium AB and CD.

$$\begin{aligned}\text{Area of trapezium ABCD} &= \frac{1}{2} \text{ area of parallelogram RQCB [Fig 6 (b)]} \\ &= \frac{1}{2} \times (AB + CD) \times \text{height} \\ &= \frac{1}{2} \times (a + b) \times h\end{aligned}$$

#### Learning Outcomes

The students learn to obtain a parallelogram by appropriately juxtaposing two identical trapezia and obtain a simple insight into the formula for the area of a trapezium.

#### Remark

The teacher may encourage students to provide a proof that RBCQ is a parallelogram.

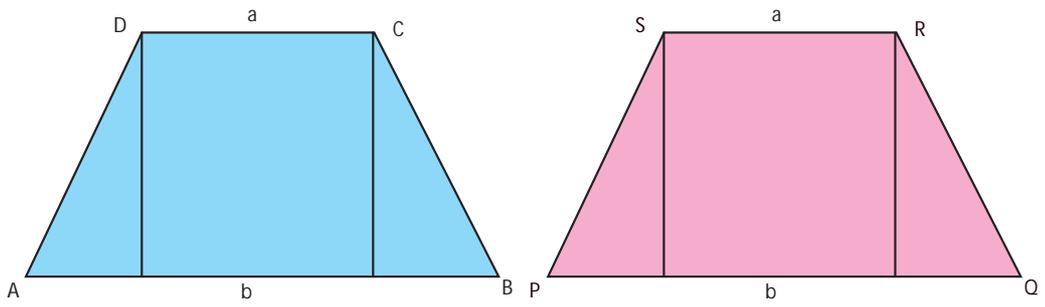


Fig 6(a)

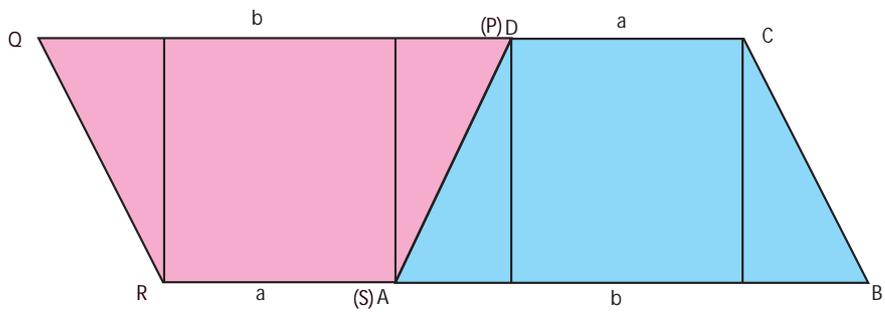
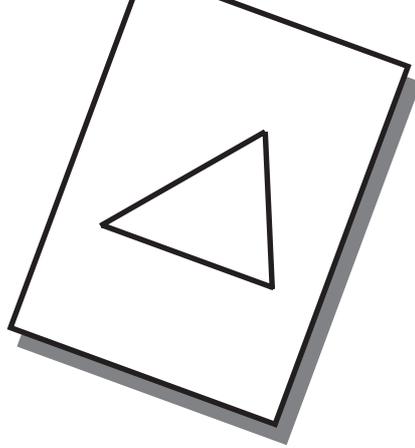


Fig 6(b)

## Activity 7



# Mid Point Theorem

### Objective

To verify the mid point theorem for a triangle, using paper cutting and pasting.

### Pre-requisite knowledge

Two lines are parallel if for a transversal cutting them, the corresponding angles are equal.

### Material Required

Coloured paper, a pair of scissors, gum.

### Procedure

From a sheet of paper, cut a triangle  $ABC$ . Find the mid points  $P$  and  $Q$  of  $AB$  and  $AC$  respectively by paper folding. Join  $P$  and  $Q$  by folding and making a crease  $PQ$ . [Fig 7 (a)] Cut  $APQ$ . Superimpose  $AQ$  over  $QC$  so that  $QP$  falls along  $CB$  as shown in Fig 7 (b).

### Observations

1. Angle  $APQ$  is now renamed as  $(A)(P)(Q)$ .  $A$  falls on  $Q$  since  $Q$  is the mid point of  $AC$ .
2. Triangle  $AQP$  is superimposed on triangle  $QCB$  and the two angles are seen to be equal. They are the corresponding angles made on  $PQ$  and  $BC$  by  $AC$ .
3. Therefore,  $PQ$  is parallel to  $BC$ .
4. Also  $(P)$  is seen to be the mid point of  $BC$  by paper folding method already described.

### Learning Outcome

Line segment joining the mid points of any two sides of a triangle is parallel to the third side and is equal to half of it.

### Remark

The exercise can be tried for any two sides of the given triangle, and for different types of triangles (acute, obtuse and right angle triangles).

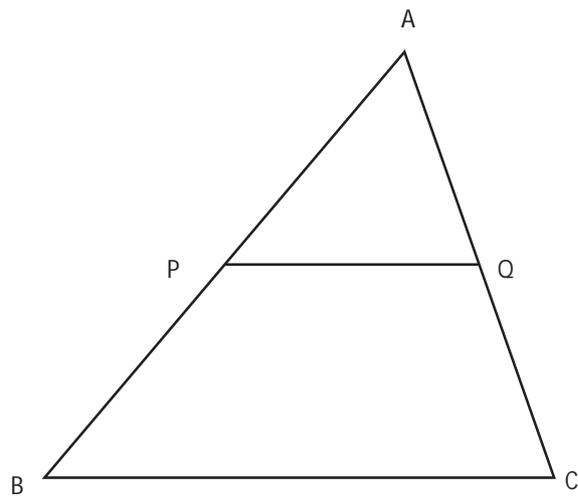


Fig 7(a)

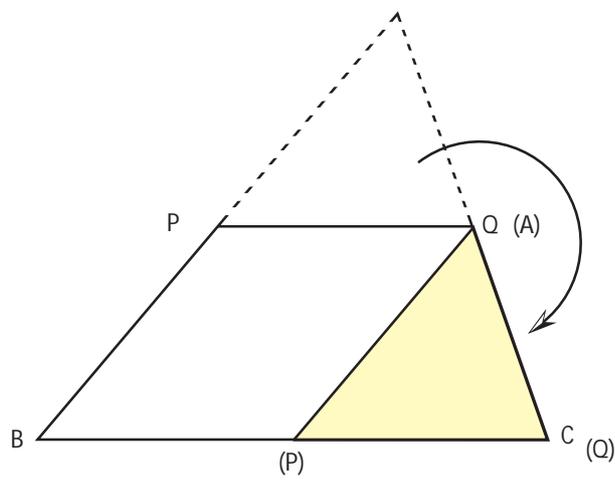


Fig 7(b)

## Activity 8

1/10	2/10	3/10	4/10	5/10
1/11	2/11	3/11	4/11	5/11
1/12	2/12	3/12	4/12	5/12
1/13	2/13	3/13	4/13	5/13

# Intercepts of equidistant parallel lines

### Objective

To divide a given strip of paper into a specified number of equal parts using a ruled graph paper.

### Pre-requisite knowledge

1. Measuring length.
2. Intercepts of equidistant parallel lines are equal.

### Material Required

Coloured paper, a pair of scissors, gum, ruled/graph paper.

### Procedure

1. Take a strip of paper say 25 cm.
2. Practice making two, three and four equal parts of the strip by the method of paper folding.
3. To make 7 equal parts
  - a) Take a ruled paper.
  - b) Give numbers ( 0, 1, 2, ... ) to the equidistant parallel lines. [Fig 8 (a)]
  - c) Keep the starting point of the strip on zero and place the end point of the strip on a line numbered 7 or multiple of 7. [Fig 8 (b)]
  - d) As shown in Fig 8 (a), the strip is arranged between 0 and 14, parallel equidistant lines.
  - e) Marking a point on every second line which intersects the strip divides the strip into seven equal parts.
4. To make any number of equal parts, repeat the procedure and place the end point of the strip on the equidistant parallel line equal to the number of parts required or a multiple of that number.

### Observations

1. The students observe that making 2, 3, 4, 5, 6 equal parts is easy by paper folding method.
2. To make 7, 11, 13 ... equal parts it is difficult by measuring length and dividing it to given equal parts.
3. Students find this activity as an interesting application of the property they have learned in the class.

### Learning Outcomes

1. Students learn how to divide a strip of paper in any number of equal parts.
2. They learn to apply the property of equidistant parallel lines.

**Remark**

This activity can be extended as follows

1. Take 15 equal strips of equal length and breadth.
2. Make 2, 3, 4, ... 15 equal parts of these strips by methods explained above.
3. Stick them on any cardboard. [Fig 8 (b)]
4. This is called as fraction chart.
5. Ask students to observe this chart and write their observations.

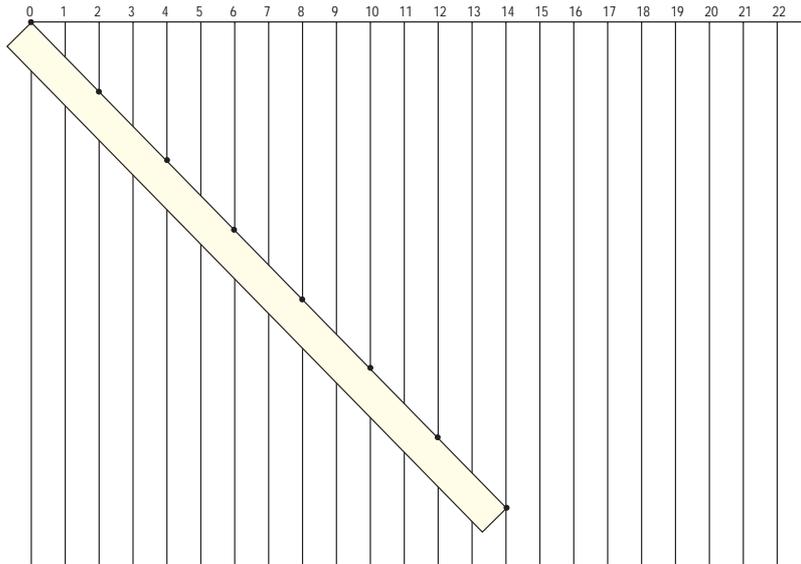


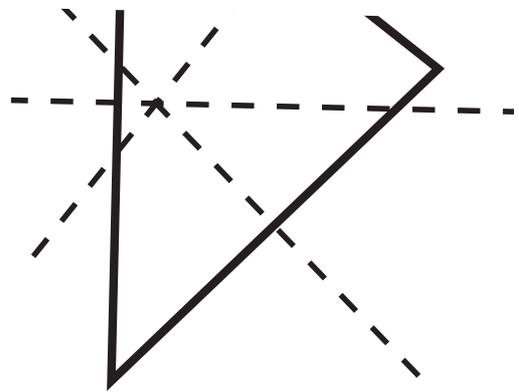
Fig 8 (a)

							1/2																
				1/3					2/3														
			1/4				2/4				3/4												
		1/5			2/5			3/5			4/5												
	1/6		2/6		3/6		4/6		5/6														
	1/7		2/7		3/7		4/7		5/7		6/7												
	1/8		2/8		3/8		4/8		5/8		6/8		7/8										
	1/9		2/9		3/9		4/9		5/9		6/9		7/9										
	1/10		2/10		3/10		4/10		5/10		6/10		7/10		8/10								
	1/11		2/11		3/11		4/11		5/11		6/11		7/11		8/11		9/11						
	1/12		2/12		3/12		4/12		5/12		6/12		7/12		8/12		9/12		10/12				
	1/13		2/13		3/13		4/13		5/13		6/13		7/13		8/13		9/13		10/13		11/13		
	1/14		2/14		3/14		4/14		5/14		6/14		7/14		8/14		9/14		10/14		11/14		12/14

Fig 8 (b)

## Activity 9

# Circumcentre of a triangle



### Objective

To illustrate that the perpendicular bisectors of the sides of a triangle concur at a point (called the circumcentre) and that it falls

- inside for an acute-angled triangle.
- on the hypotenuse of a right-angled triangle.
- outside for an obtuse-angled triangle.

### Pre-requisite knowledge

Familiarity with Activity 1A.

### Material Required

Coloured paper, pencil, a pair of scissors, gum.

### Procedure

1. Cut an acute angled triangle from a coloured paper and name it as ABC.
2. Form the perpendicular bisector EF of AB using paper-folding method.
3. Similarly get the perpendicular bisectors GH and IJ of the sides AC and BC respectively.
4. Repeat the activity for right and obtuse angled triangles.

### Observations

1. The students see that the three perpendicular bisectors (the three creases obtained) are concurrent.
2. For the acute angled triangle, the circumcentre lies inside the triangle as shown in Fig 9 (a).
3. For the right angled triangle, the circumcentre is the mid point of the hypotenuse as shown in Fig 9 (b)
4. For the obtuse angled triangle, the circumcentre lies outside the triangle as shown in Fig 9 (c).

### Learning Outcomes

1. The circumcentre is equidistant from the three vertices of the triangle. Hence a circle can be drawn passing through the three vertices with circumcentre as the center. This circle is called circumcircle.
2. The perpendicular bisectors of the sides of a triangle can never form a triangle since they pass through a point.

### Remark

The teacher may encourage the students to provide a proof of concurrence and of the observation about the location of the circumcentre.

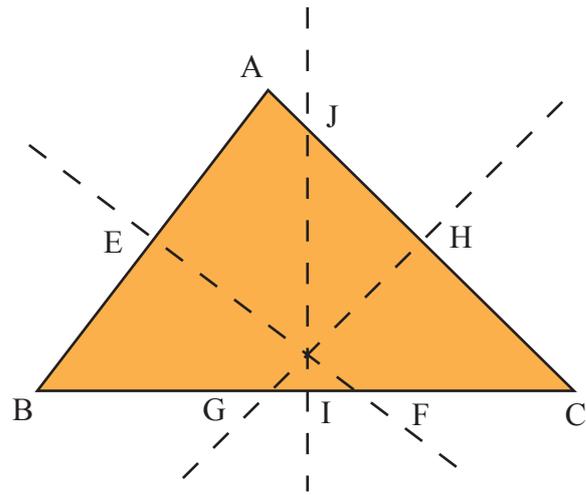


Fig 9 (a)

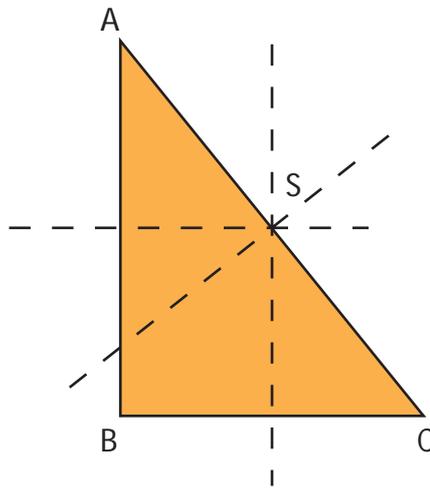


Fig 9 (b)

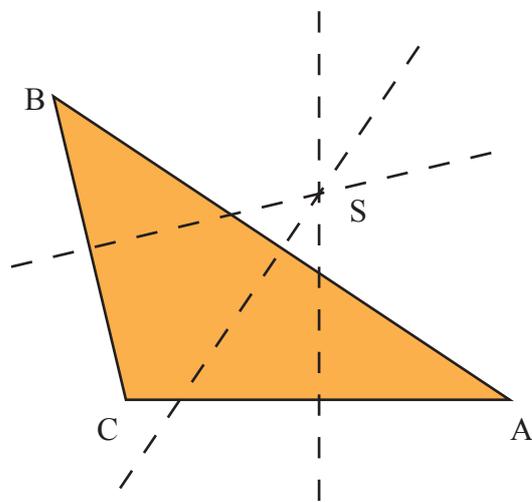
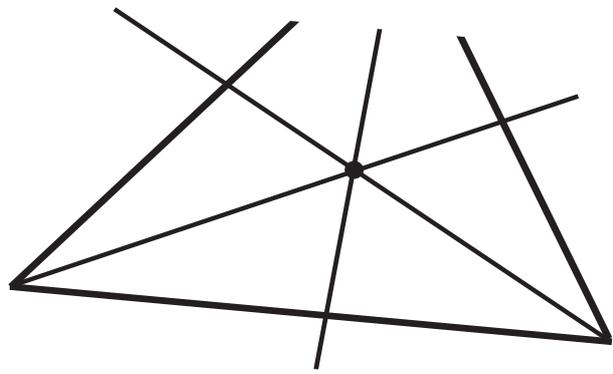


Fig 9 (c)

## Activity 10



### Incentre of a triangle

#### Objective

To illustrate that the internal bisectors of the angles of a triangle concur at a point (called the incentre), which always lies inside the triangle

#### Pre-requisite knowledge

Familiarity with Activity 1A.

#### Material Required

Coloured papers, fevicol and a pair of scissors.

#### Procedure

1. Cut an acute angled triangle from a colored paper and name it as PQR.
2. Fold along the vertex P of the triangle in such a way that the side PQ lies along PR.
3. The crease thus formed is the angle bisector of angle P. Similarly get the angle bisectors of angle Q and R. [Fig 10 (a)]
4. Repeat the same activity for a right angled triangle and obtuse angled triangle. [Fig 10 (b) and Fig 10 (c)]

#### Observations

1. We see that the three angle bisectors are concurrent and the point is called the incentre (I).
2. We observe that the incentre of an acute, an obtuse and right angled triangle always lies inside the triangle.

#### Learning Outcomes

1. The incentre I is equidistant from three sides of the triangle. Hence, a circle can be drawn touching all the sides, with I as its center. This circle is called In-circle.
2. The angle bisectors of a triangle can never form a triangle since they pass through a point.

#### Remark

The teacher may encourage the students to provide a proof of concurrence and of the observation of the location of the incentre.

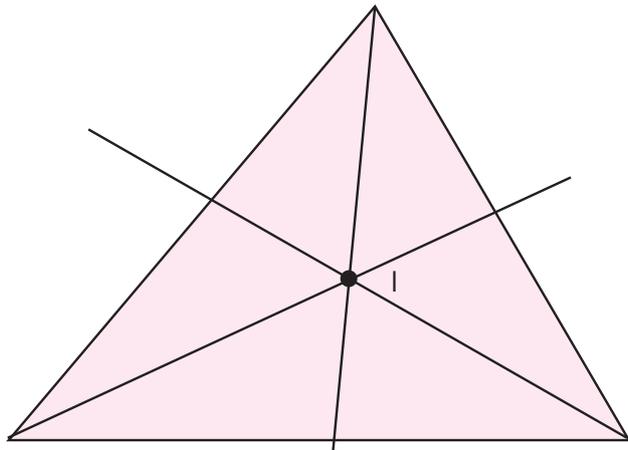


Fig 10(a)

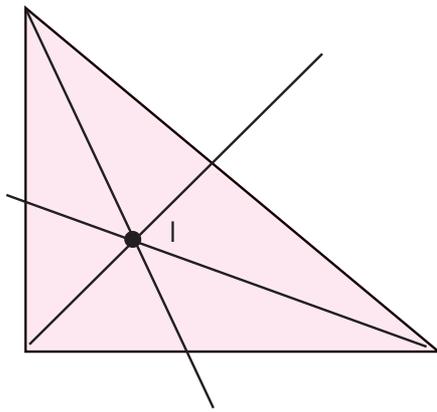


Fig 10(b)

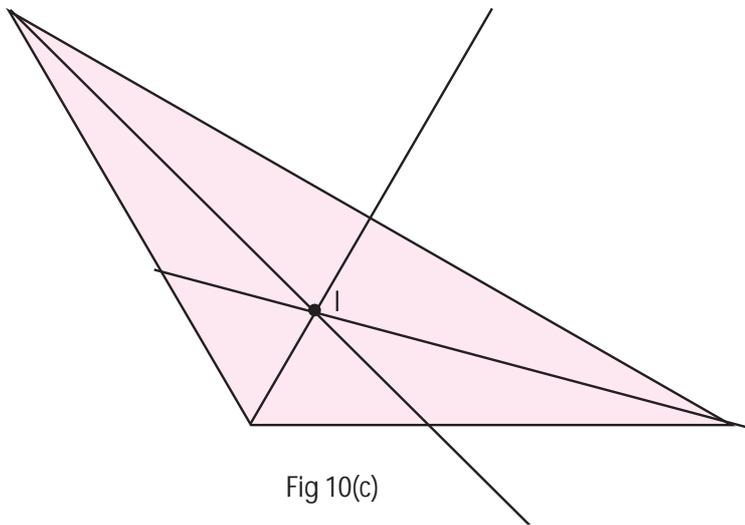
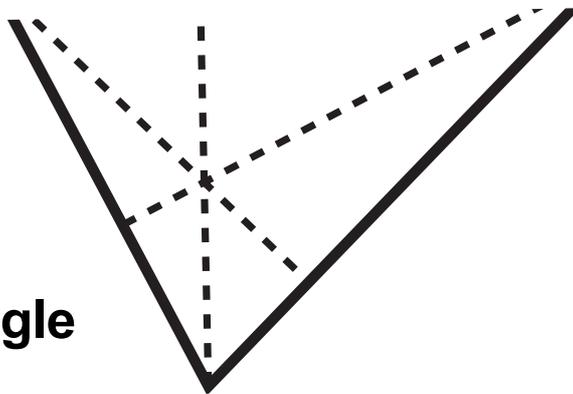


Fig 10(c)

## Activity 11

# Orthocentre of a triangle



### Objective

To illustrate that the altitudes of triangle concur at a point (called the orthocentre) and that it falls

- inside for an acute angled triangle.
- at the right angle vertex for a right angled triangle.
- outside for an obtuse angled triangle.

### Pre-requisite knowledge

Familiarity with Activity 1A.

### Material Required

Coloured papers, pencil, a pair of scissors, gum.

### Procedure

1. Take three rectangular sheets of paper and draw three types of triangles on each of the sheet: acute angled, right angled and obtuse angled.
2. For the acute angled triangle, fold the perpendicular through the vertex to the opposite side. This is one of the altitudes.
3. Make similar folds to get the other two altitudes. Locate the point of intersection of the altitudes.
4. Repeat the same activity for right and obtuse angled triangles.

### Observations

1. The students observe that the three altitudes of a triangle are concurrent. This point is called the orthocentre (O).
2. For the acute angled triangle, the orthocentre lies inside the triangle as shown in Fig 11 (a).
3. For the right angled triangle, the orthocentre is the vertex of the right angle as shown in Fig 11 (b).
4. For the obtuse angled triangle, the orthocentre lies outside the triangle as shown in Fig 11 (c).

### Learning Outcomes

Students learn that the altitudes of the sides of a triangle can never form a triangle since they pass through a point.

### Remark

The teacher may encourage the students to provide a proof of the concurrence of altitudes and of the observation of the location of the orthocentre.

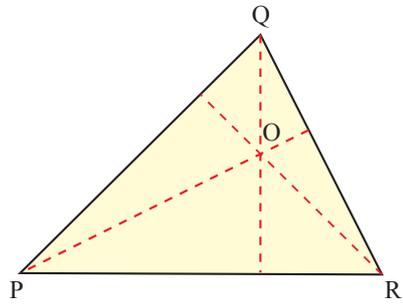


Fig 11 (a)

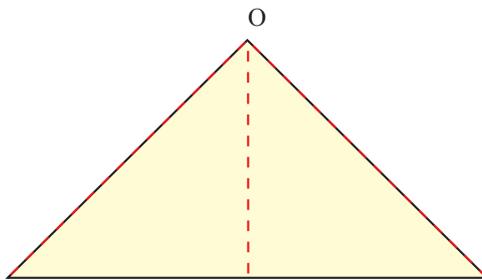


Fig 11 (b)

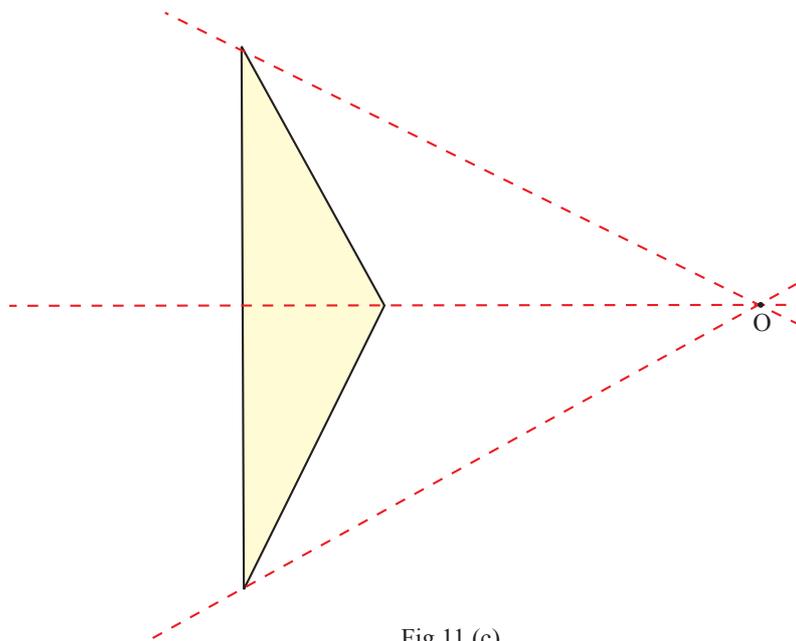
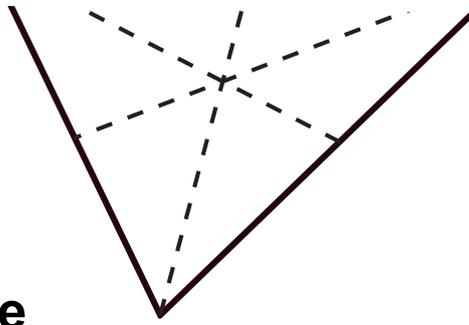


Fig 11 (c)

## Activity 12

# Centroid of a triangle



### Objectives

To illustrate that the medians of a triangle concur at a point (called the centroid), which always lies inside the triangle.

### Pre-requisite knowledge

Familiarity with Activity 1A.

### Materials required

Coloured paper, pencil, a pair of scissors, gum.

### Procedure

1. From a sheet of paper, cut out three types of triangle: acute-angled triangle, right-angled triangle and obtuse-angled triangle.
2. For an acute-angled triangle, find the mid-points of the sides by bringing the corresponding two vertices together. Make three folds such that each joins a vertex to the mid-point of the opposite side. [Fig 12 (a)]
3. Repeat the same activity for a right-angled triangle and an obtuse-angled triangle. [Fig 12 (b) and Fig 12 (c)]

### Observations

The students observe that the three medians of a triangle concur. They also observe that the centroid of an acute, obtuse or right-angled triangle always lies inside the triangle.

### Learning Outcomes

1. The students learn that the medians of a triangle are concurrent and cannot form a triangle.
2. The students will learn that the centroid is the point of the trisection of the median of a triangle.

### Remark

The teacher may encourage the student to provide a proof of concurrence and of the observation about the location of the centroid.

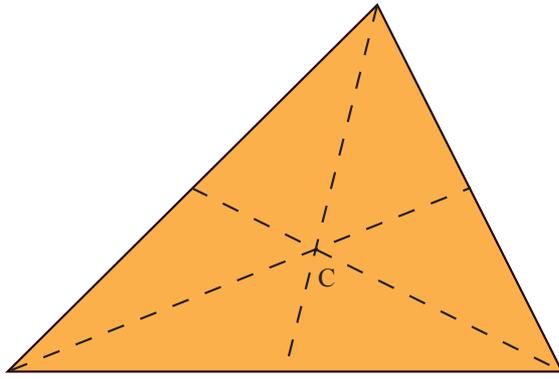


Fig 12 (a)

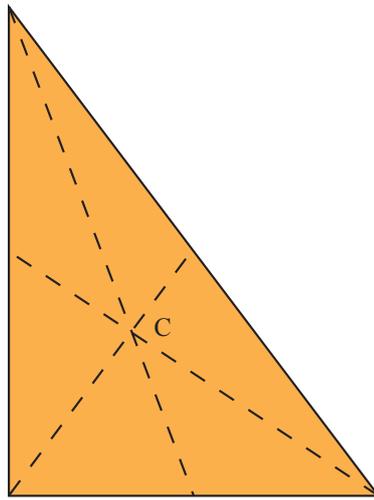


Fig 12 (b)

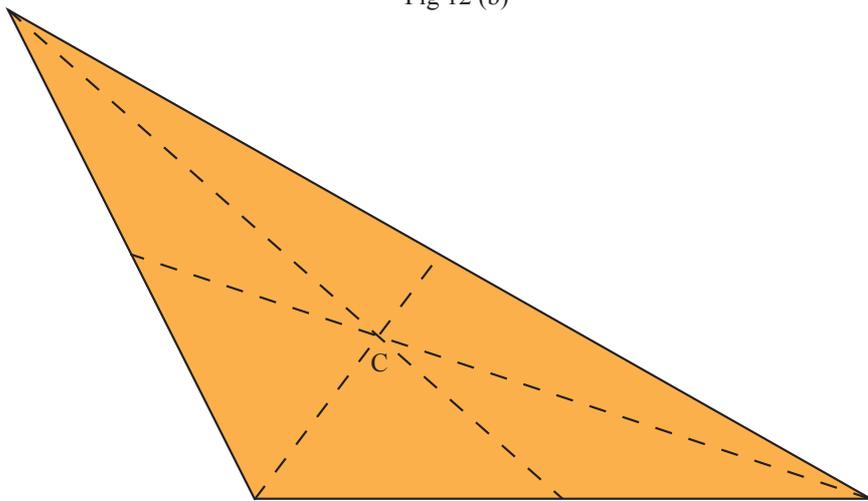
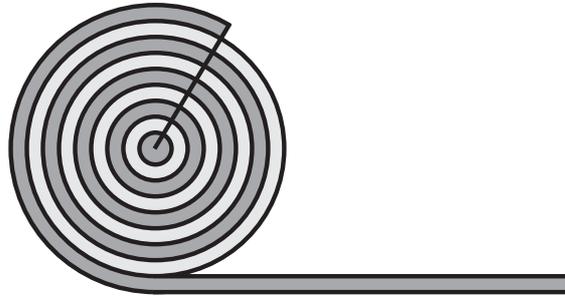


Fig 12 (c)

## Activity 13A



### Area of a circle

#### Objective

To give suggestive demonstration of the formula that the area of the circle is half the product of its circumference and radius.

#### Pre-requisite knowledge

Formula for the circumference of a circle.

Formula for the area of a triangle.

#### Material Required

Coloured thread, a pair of scissors.

#### Procedure

1. Draw a circle of a certain radius say  $r$ .
2. Fill up the circle with concentric circles made of different coloured threads as shown in Fig 13A (a) so that there is no gap left in between. Obviously, the smallest circle will be a point circle.
3. Cut off the circle formed by threads along the radius of the circle starting from a point 'O'.
4. Open all the threads, arrange them from the smallest to the longest forming a triangle.

#### Observations

1. Area of the circle is same as the area of triangle.
2. The triangle so formed has the base equal to the circumference of the outermost circle, which is  $2\pi r$ . Height of the triangle is equal to the radius of the circle. Thus the area of the circle is  $2\pi r \times r/2 = \pi r^2$

#### Learning Outcomes

The students express the area of a circle in terms of the area of a more elementary figure, namely a triangle, and thus build a geometrical intuition of the formula  $\pi r^2$  for the area of circle.

#### Remark

The teacher may point out the fact that this is not a rigorous mathematical proof for the formula for area of a circle.

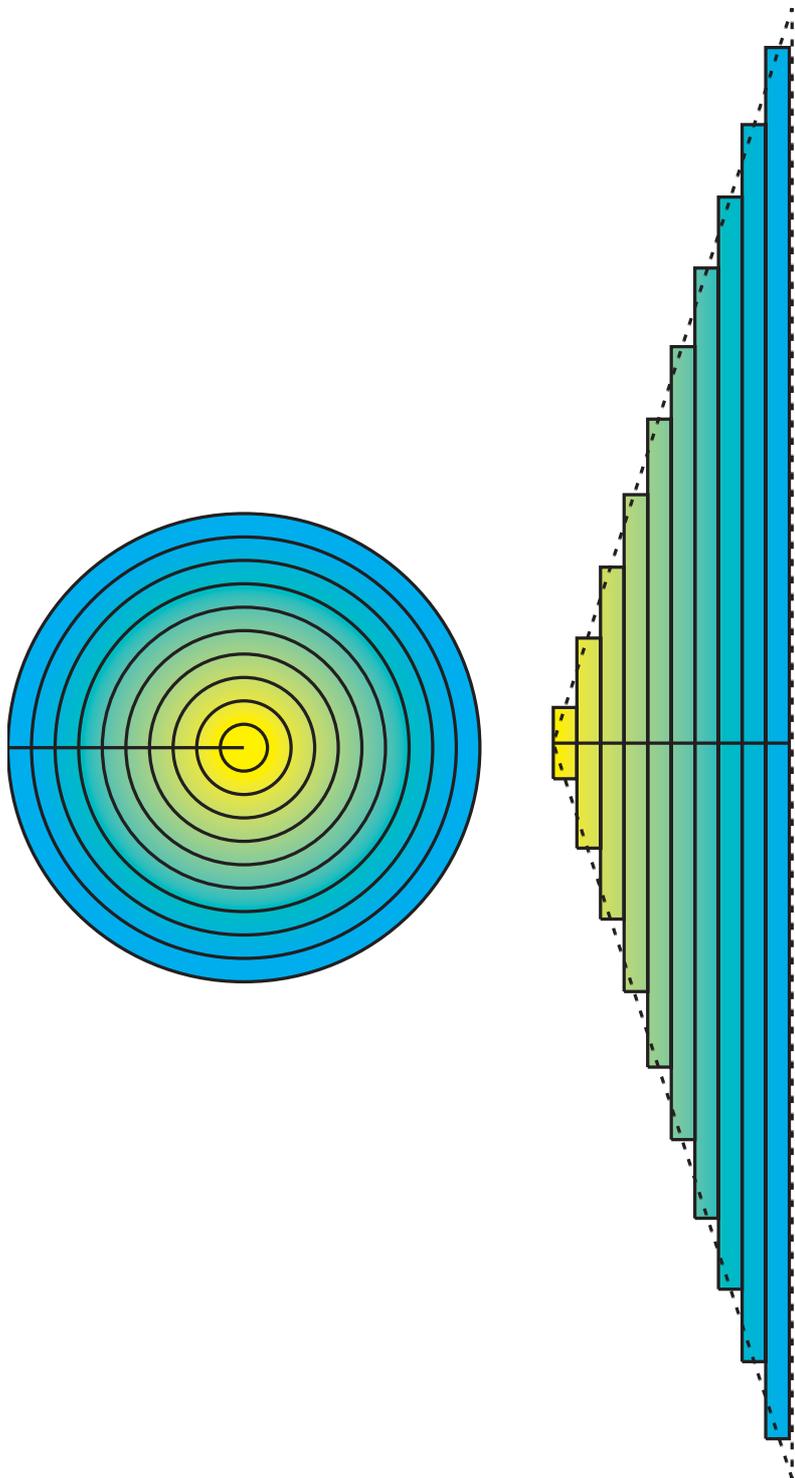
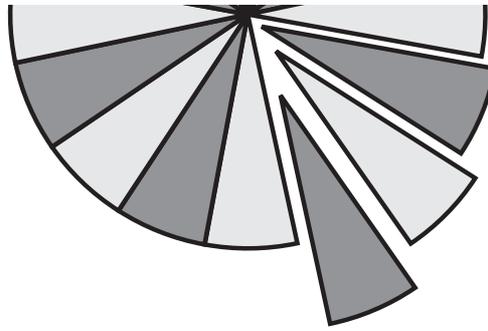


Fig 13A (a)

## Activity 13B



### Area of a circle

#### Objective

To give a suggestive demonstration of the formula that the area of circle is half the product of its circumference and radius.

#### Pre-requisite knowledge

1. Formula for the circumference of a circle.
2. Formula for the area of a rectangle.

#### Material Required

Coloured paper, compass, scale, a pair of scissors, gum, colours.

#### Procedure

1. Draw a circle of radius  $r = 4$  cm (say) on the paper.
2. Divide the circle into 16 equal parts. [Fig 13B (a)]
3. Cut all the 16 parts and arrange them to get the Fig 13B (b).
4. Take any part from any side and further divide it into 2 parts. [Fig 13B (c)]
5. To complete the shape of rectangle arrange these two smaller parts at the corners of the shape obtained in Fig 13B (b).
6. Find the length and the breadth of the rectangle so formed [Fig 13B (d)].
7. Find the area of the rectangle.

#### Observations

1. The students will observe that the area of rectangle  $[(2\pi r/2) \times r] = \pi r^2$
2. The students will observe that the rectangle is obtained from parts of circle.  
Hence area of circle =  $\pi r^2$

#### Learning Outcomes

1. The students will learn the skill of transforming one geometrical shape into another.
2. They will also learn the elementary idea of approximation by transforming a circle into a rectangle like figure.
3. They will also learn that approximation becomes better and better as the number of parts increases.
4. The students express the area of a circle in terms of the area of a more elementary figure, namely a rectangle, and thus build a geometrical intuition of the formula  $\pi r^2$  for the area of circle.

**Remark**

1. Teacher may ask the students to cut the circle into greater number of equal parts. Eg. 32, 64, ... So as to convert the circle into a figure which appears more and more like a rectangle.
2. The teacher may point out the fact that this is not a rigorous mathematical proof for the formula for area of a circle.

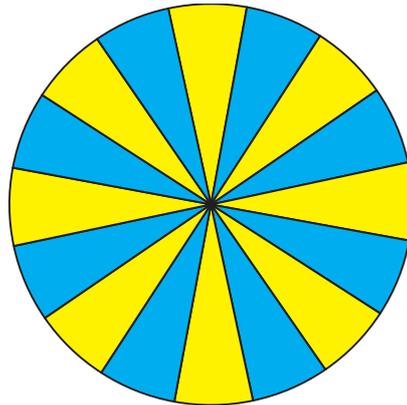


Fig 13B (a)

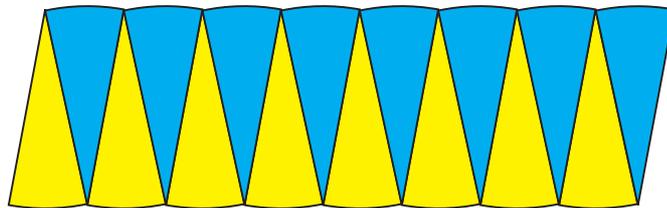


Fig 13B (b)

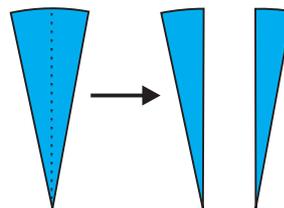


Fig 13B (c)

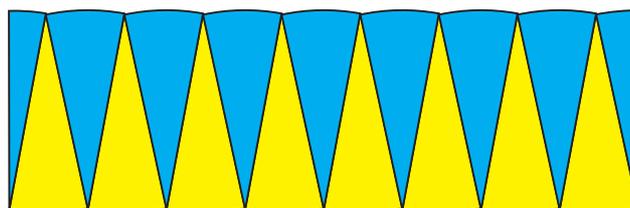
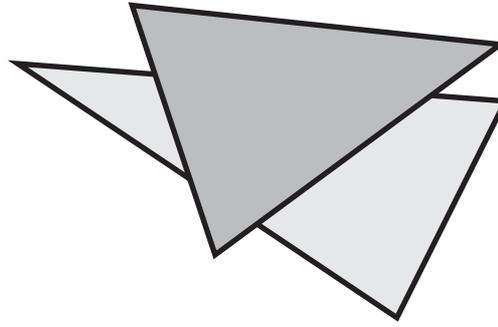


Fig 13B (d)

## Activity 14



# Triangle inequality

### Objectives

1. To verify that the sum of any two sides of a triangle is always greater than the third side.
2. To verify that the difference of any two sides of a triangle is always less than the third side.

### Pre-requisite knowledge

Measurements and comparison of line segments.

### Material Required

Chart paper, pencil, ruler and broom sticks.

### Procedure

Get sticks of different lengths. Take three at a time. For example:

Set I (4cm, 5cm, 10cm)  
(3cm, 5cm, 9cm)  
(5cm, 6cm, 14cm)

Set II (5cm, 5cm, 10cm)  
(6cm, 6cm, 12cm)  
(4cm, 4cm, 8cm)

Set III (5cm, 6cm, 10cm)  
(7cm, 8cm, 10cm)  
(8cm, 9cm, 14cm)

For each triplet of numbers in a given set above, try to form triangle.

### Observations

Observe the lengths that form the triangles. See Figs 14 (a), 14 (b) and 14 (c). The students will notice that a triangle is possible only if the sum of any two sides of a triangle is greater than the third side. In each possible case, they will notice that the difference of any two sides is less than the third.

### Learning Outcomes

The students learn that with any three line segments, you cannot always construct a triangle. The given lengths must satisfy the condition that (a) the sum of any two sides of a triangle is always greater than the third side and (b) the difference of any two sides is less than the third side.

### Remark

The conditions have to be satisfied for every pair of sides of a triangle. Also if condition (a) is satisfied (b) is automatically satisfied and vice-versa.

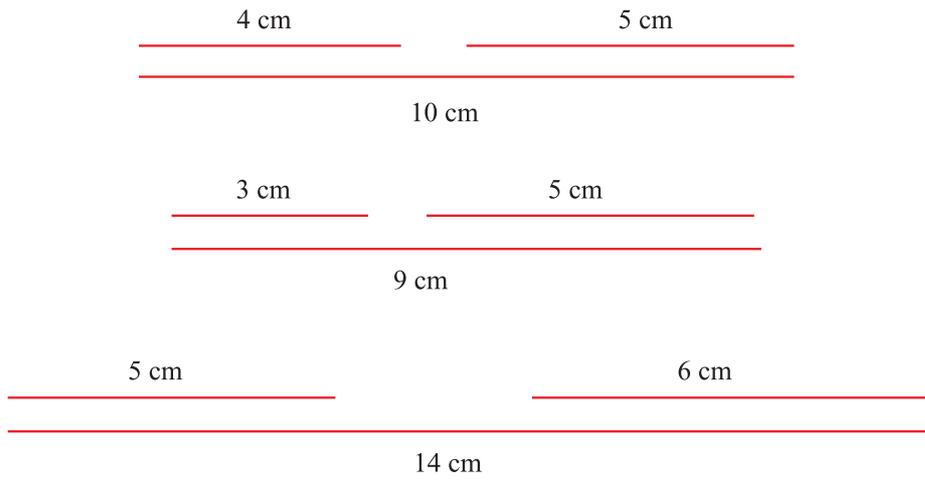


Fig 14 (a)

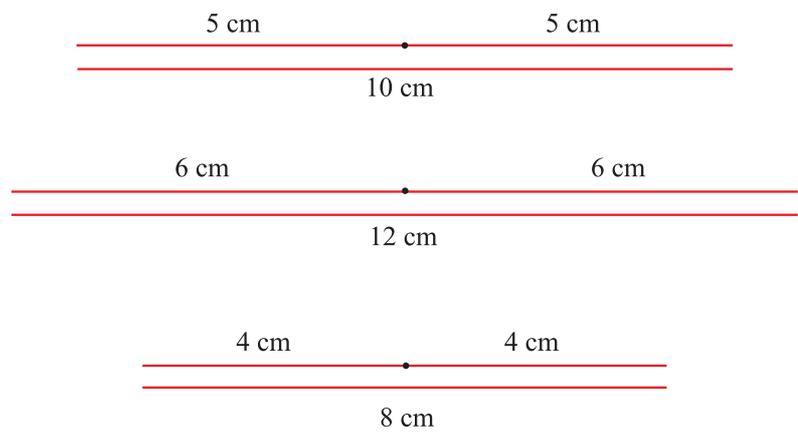


Fig 14 (b)

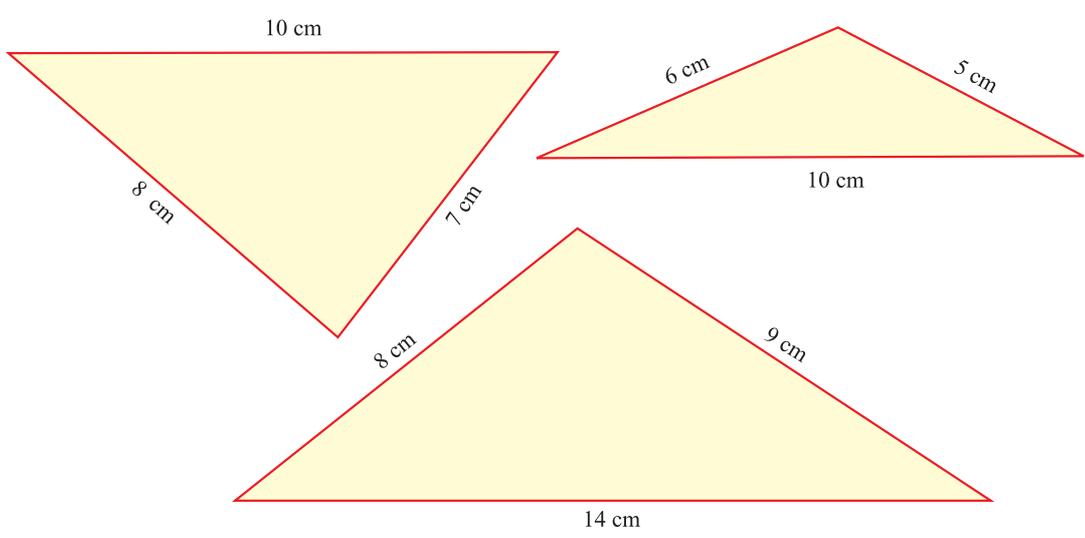
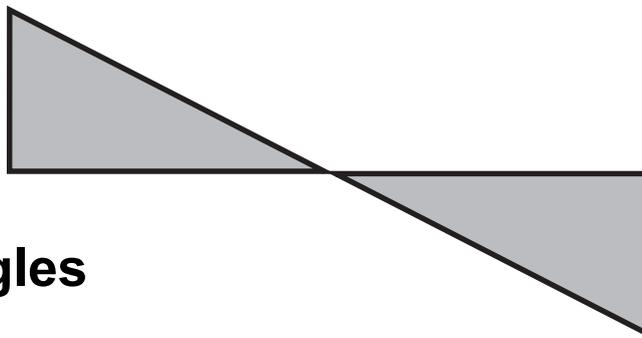


Fig 14 (c)

## Activity 15



# Congruent triangles

### Objectives

To explore criteria for congruency of triangles, using a set of triangular cut-outs.

### Pre-requisite knowledge

Idea of congruent triangles.

### Materials required

Different sets of triangular cut-out.

### Procedure

1. Measure the sides and angles of the given sets of triangles.
2. Using the various criteria of congruency, take out the pairs of triangle which are congruent.
3. Record the observations.
4. Take the set of triangles in which all the three pairs of corresponding angles are equal and check whether they are congruent.
5. Record the observations.
6. Take the set of triangles in which two pairs of sides are equal and a pair of corresponding angles (not the included angle) are equal. Check whether they are congruent.
7. Record the observations.

### Observations

1. The students will list the sets of triangles which are congruent and the criteria of congruency used for this purpose.
2. The students will observe that there is no AAA congruency criteria for the triangles.
3. The students will also observe that there is no SSA congruency criteria for the triangles.

### Learning outcome

The students will learn that only certain sets of criteria involving the equality of sides and angles of triangles lead to their congruency.

### Remarks

1. Teacher may ask the students to find more examples in which AAA, SSA are not the criterion of congruency in triangles.
2. The teacher may emphasize the fact that AAA is a necessary condition for congruency of triangles but not the sufficient condition to prove the congruency of triangles.
3. Fig 15 (a) provides a sample of triangles of various dimensions.

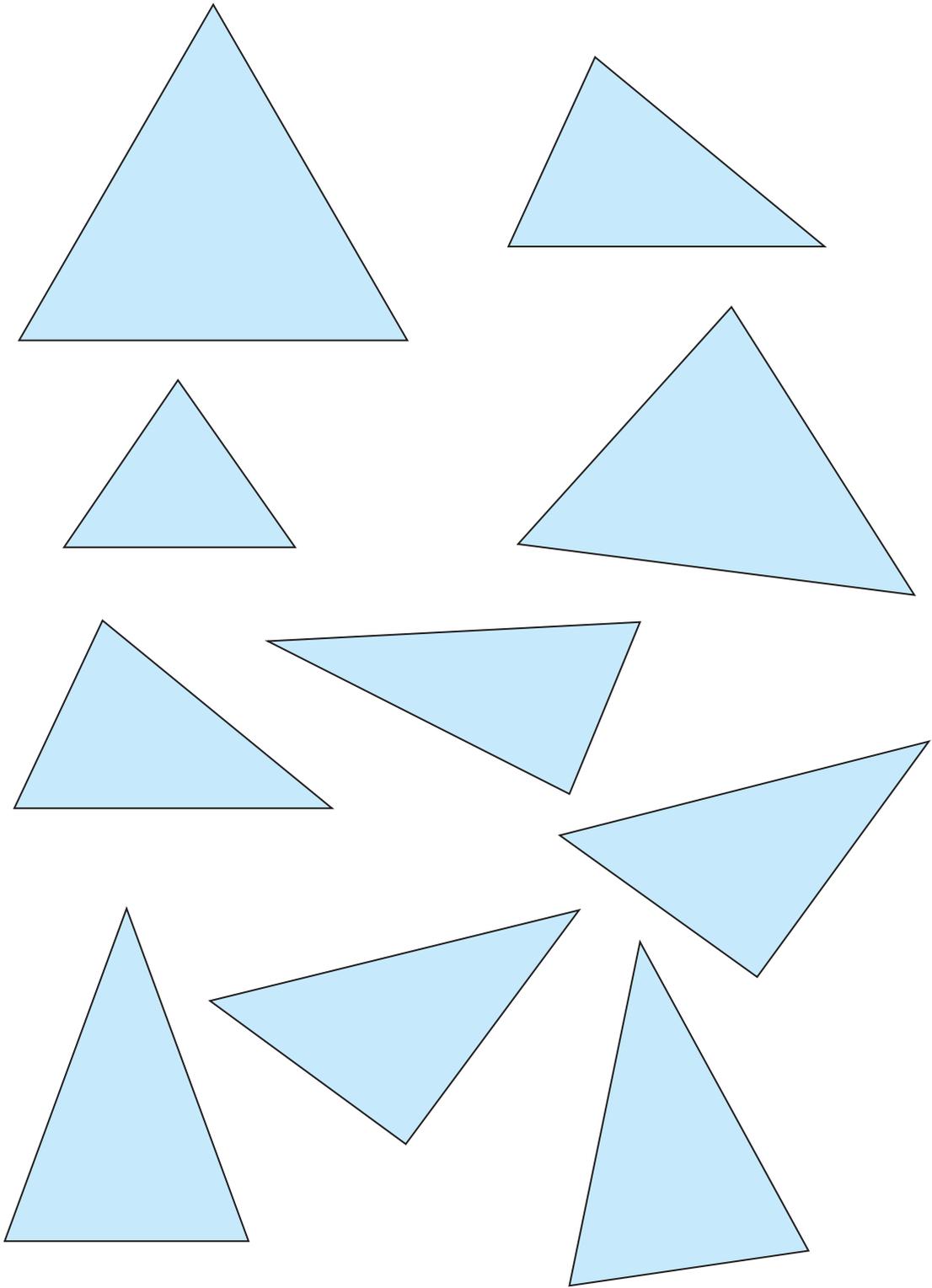
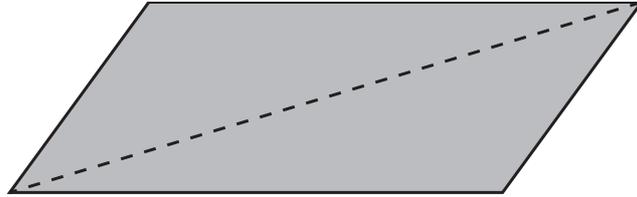


Fig 15 (a)

## Activity 16&17



# Properties of parallelogram

### Objective

To explore similarities and differences in the properties with respect to diagonals of the following quadrilaterals – a parallelogram, a square, a rectangle and a rhombus.

### Pre-requisite knowledge

1. Construction of the diagram of parallelogram, rhombus, square and rectangle.
2. Knowledge of properties of sides and angles of the above mentioned figures.

### Material Required

Chart papers, pencil, compass, scale and a pair of scissors.

**Property 1:** A diagonal of a parallelogram divides it into two congruent triangles.

### Procedure

1. Make a parallelogram on a chart paper and cut it.
2. Draw diagonal of the parallelogram [Fig 16&17(a)].
3. Cut along the diagonal and obtain two triangles.
4. Superimpose one triangle onto the other [Fig 16&17(b)].

### Observation

Two triangles are congruent to each other.

### Learning Outcome

Students would be able to infer that diagonal always divides the parallelogram into two triangles of equal areas.

### Remark

Repeat the same activity with the other diagonal of the parallelogram.

**Property 2 :** Diagonals of a parallelogram bisect each other.

### Procedure

1. Draw the parallelogram and its both diagonals.
2. Cut the four triangles formed. Name them 1, 2, 3 and 4 [Fig 16%17(c)].
3. Observe that triangle 2 is congruent to triangle 4 and triangle 1 is congruent to triangle 3 by superimposing them on each other.

### Observations

1. Base of triangle 2 = Base of triangle 4
2. Base of triangle 1 = Base of triangle 3
3. Thus the diagonals bisect each other.

### Learning Outcome

Students can also infer that vertically opposite angles are equal.

### Remark

1. Teacher can ask the students to check the congruency of the triangle 1 and triangle 4. Why these triangles are not congruent?
2. Teacher can ask the students to repeat the same activities with rhombus, square and rectangle to find the properties of diagonals.
3. The students can explore when diagonals bisect each other at right angles and other properties (using elementary methods of paper folding described in activity 1A).
4. Students should summarize the results in the following format.

Sr. No.	Properties	Parallelogram	Square	Rectangle	Rhombus
1	Diagonals bisect each other	yes	yes	yes	yes
2	Diagonals are perpendicular to each other				
3	Diagonals have equal length				
4	Diagonal divides the given quadrilateral into two congruent triangles				

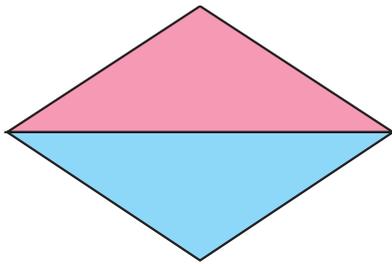


Fig 16 & 17 (a)

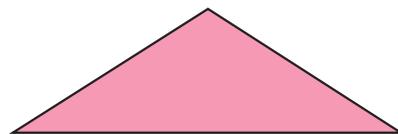


Fig 16 & 17 (b)

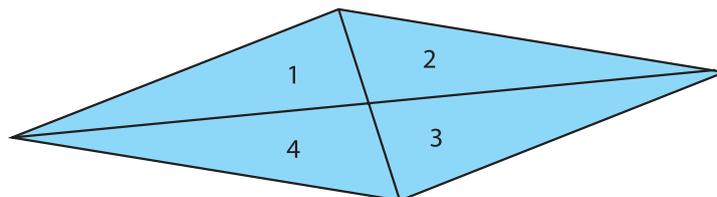
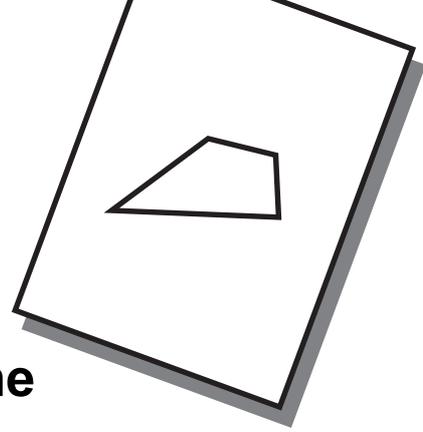


Fig 16 & 17 (c)

## Activity 18



# The quadrilateral formed by the mid points of a quadrilateral

### Objective

To show that the figure obtained by joining the mid-points of consecutive sides of the quadrilateral is a parallelogram.

### Pre-requisite knowledge

1. Finding mid-points of the line segments by paper folding. (Familiarity with Activity 1A)
2. If in a quadrilateral a pair of opposite sides are equal and parallel, then it is a parallelogram.

### Material required

Coloured paper, a pair of scissors, gum.

### Procedure

Cut off a quadrilateral ABCD of paper with prescribed dimensions. Mark the mid-points P, Q, R and S of the sides AB, BC, CD and DA respectively by folding the sides appropriately. Cut off the quadrilateral PQRS. [Fig 18 (a)]

### Observations

By considering triangle ABC, it follows that PQ the line joining the mid-points of AB and BC is parallel to AC and  $PQ = \frac{1}{2} AC$  (Mid-point theorem). Similarly from triangle ADC,  $RS = \frac{1}{2} AC$  and RS is parallel to AC.

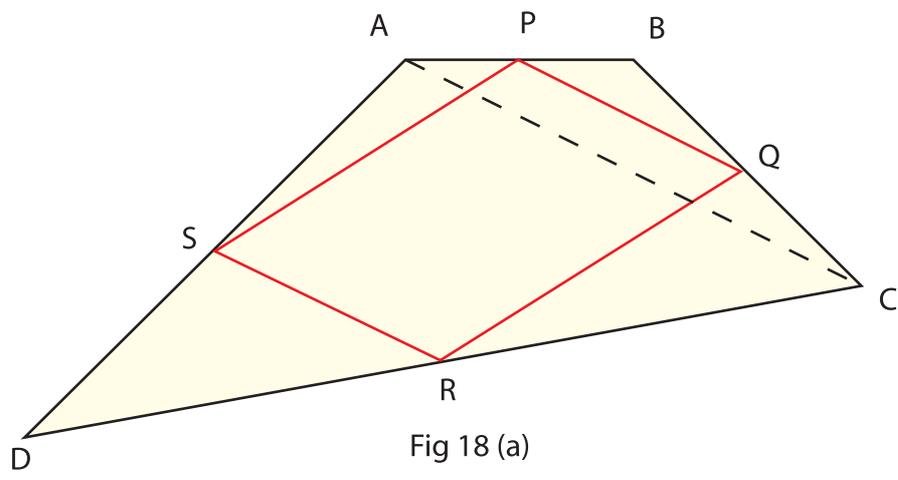
PQ is parallel to SR and  $PQ = SR$ , so PQRS is a parallelogram.

### Learning outcome

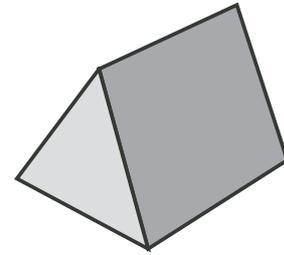
The students learn that a parallelogram can be obtained from any quadrilateral by joining the mid-points of its sides.

### Remark

The students can compare the areas of the parallelogram PQRS and the quadrilateral ABCD.



## Activity 19



# Area of a right triangular prism and a pyramid

### Objective

To make the net for a right triangular prism and a right triangular pyramid (regular tetrahedron) and obtain the formulae for the total surface area.

### Pre-requisite knowledge

1. Construction of a right triangular prism and a right triangular pyramid in terms of their faces.
2. Understanding of the terms lateral surface area and total surface area.
3. Formulae for the area of a rectangle and a triangle.

### Material Required

Chart paper, scale, pencil, a pair of scissors and gum.

### Procedure

#### A) Right triangular Prism

1. Draw the nets with given dimensions on a chart paper.
2. Cut out these nets.
3. Fold along the lines to form a prism. Join the edges with gum.
4. Obtain the two prisms.

### Observations

1. Observe that prism obtained from the net in Fig 19 (a) has three congruent squares as lateral surfaces and two congruent equilateral triangles as base.
2. The lateral surface area in this case =  $3 \times$  the area of square.
3. The total surface area = lateral surface area +  $2 \times$  the area of the equilateral triangle
4. Observe that the prism obtained from the net in Fig 19 (b) has three congruent rectangles as lateral surface and two equilateral triangles as base.
5. So, the lateral surface area in this case =  $3 \times$  the area of rectangle.  
The total surface area = lateral surface area +  $2 \times$  the area of equilateral triangle.

#### B) Right triangular pyramid (Regular tetrahedron)

1. Draw the net with given dimensions on a chart paper.
2. Cut out this net.
3. Fold along the lines to form a pyramid. Join the edges with gum.
4. Obtain the right triangular pyramid.

### Observations

Observe that the pyramid obtained from net in Fig 19 (c) has four congruent equilateral triangles, where three congruent equilateral triangles form lateral surface of pyramid and one triangle forms the base.

So the lateral surface area of pyramid =  $3 \times$  the area of equilateral triangle.

Total surface area of pyramid = lateral surface area + area of base triangle  
=  $4 \times$  the area of equilateral triangle.

### Learning Outcome

The students learn to make prisms and pyramids from nets. Further they are able to obtain lateral and total surface area in terms of the area of triangles and rectangles.

### Remark

Teachers should help the students to observe, that  
 $3 \times$  the area of rectangle = perimeter of base  $\times$  height

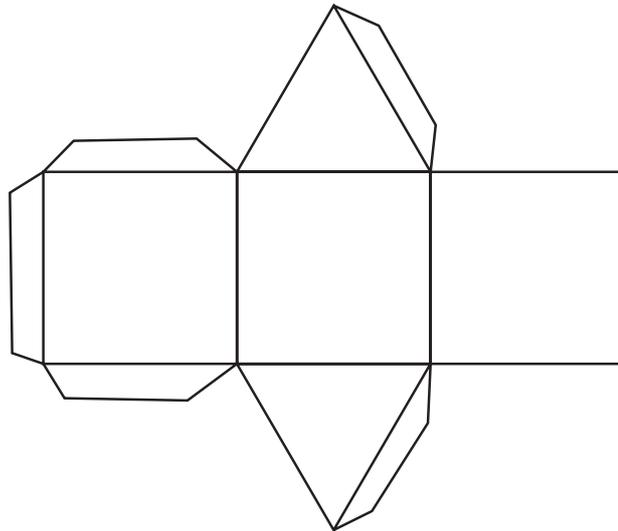


Fig 19 (a)

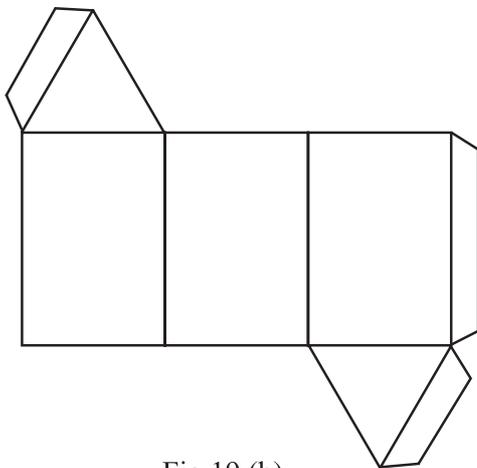


Fig 19 (b)

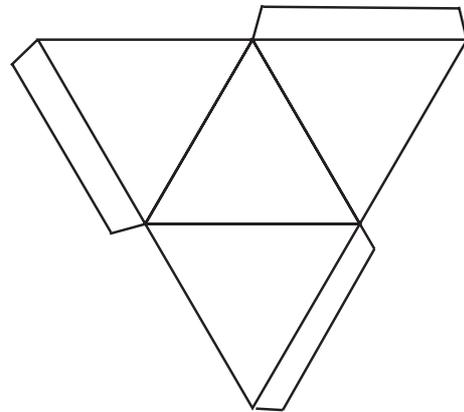
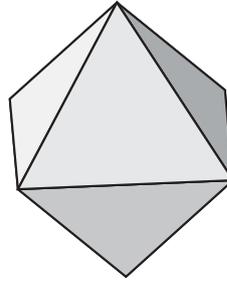


Fig 19 (c)

## Activity 20



### Euler's formula

#### Objective

To verify Euler's formula for different polyhedra : Prism, Pyramid and Octahedron.

#### Pre-requisite knowledge

1. Shape of a triangular prism, pentagonal prism, pyramid on a quadrilateral base, pyramid on a pentagonal base, pyramid on a hexagonal base and octahedron.
2. Identification of vertices, edges and faces of polyhedra.

#### Material Required

Chart paper, pencil, compass, scale, a pair of scissors, cello tape.

#### Procedure

1. Take a chart paper and draw the nets shown.
2. Fold the above nets along the lines and join with adhesive tape.
3. Obtain prism, pentagonal prism, pyramid on a quadrilateral base, pentagonal pyramid, hexagonal pyramid and octahedron.

#### Observation

Draw the following observation table and complete it with the help of polyhedra obtained.

Name of polyhedra	No. of face F	No. of vertices V	No. of edges E	$F - E + V$
(1) Prism				
(2) Pentagonal prism				
.				
.				
.				
.				

#### Learning Outcome

The students will verify Euler's formula. They will appreciate that different polyhedra have a common relation between F, E and V.

#### Remark

Euler's formula is an important result in the branch of mathematics called topology.

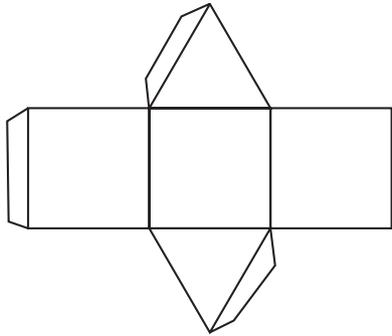


Fig 20 (a)

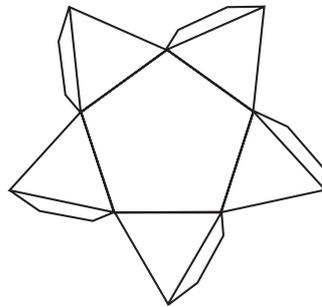


Fig 20 (b)

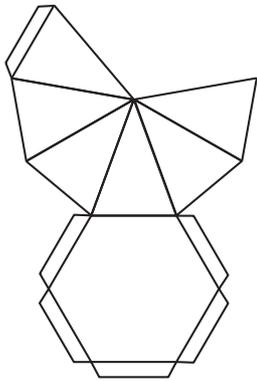


Fig 20 (c)

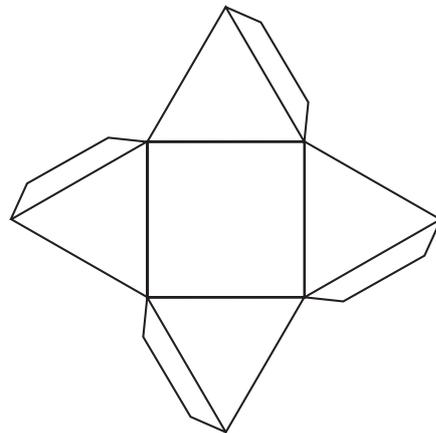


Fig 20 (d)

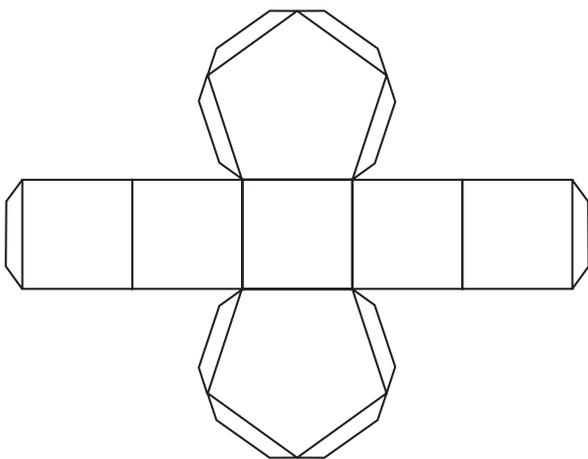


Fig 20 (e)

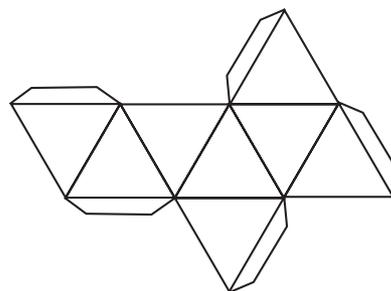
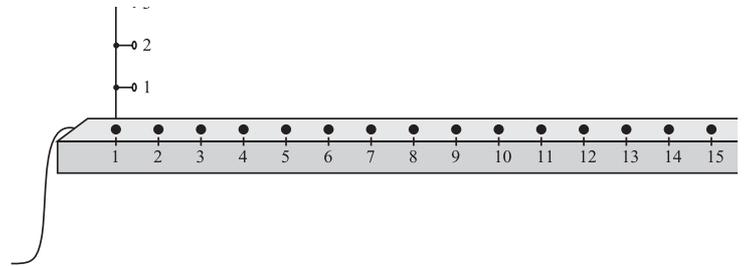


Fig 20 (f)

## Activity 21



# The square roots of natural numbers

### Objective

Obtain a line segment corresponding to the square roots of natural numbers using graduated strips.

### Pre-requisite knowledge

1. Knowledge of Pythagoras theorem, i.e., in any right angled triangle the square of the hypotenuse side is equal to the sum of the squares of the base and the perpendicular.
2. Expressing a given number as the sum of the squares of the two numbers.

### Material Required

Two wooden strips, nails, thread.

### Procedure

1. Take a wooden strip and make a scale on it (call this strip as A).
2. Make a hole on each mark as shown in Fig 21 (a).
3. Put a thread attached at the zeroth position on scale A.
4. Take another wooden strip and make a scale on it. Fix nails on it as shown in Fig 21 (b) (call this strip as B).
5. Now fix the scale B on horizontal scale A as shown in Fig 21 (c) i.e., scale A is fixed on scale B at point O.
6. For determining the line corresponding to  $\sqrt{2}$ :  
Insert scale B, in the hole 1 on scale A. Tie the thread to number one on scale B. This forms a triangle with base and height as one unit. Using Pythagoras theorem, the length of the thread is  $\sqrt{2}$ . Measure the length of the thread on scale A.

### Observations

1. The students observe that the length corresponding to  $\sqrt{2}$ , is approximately 1.41 cm.
2. They also understand that, to determine the corresponding length for  $\sqrt{13}$ , they should insert scale B into scale A at 3 and tie the thread to 2 on scale B.
3. By using Pythagoras theorem, the length of the thread is  $\sqrt{(3^2 + 2^2)} = \sqrt{13}$ .  
They can measure it on scale A; which is 3.6 cm.

### Learning Outcomes

1. The students learn to find corresponding line segment for square roots of natural numbers.
2. They can see these irrational numbers represented geometrically.

**Remark**

Teachers can take any irrational number and perform such activity for determining the line segment corresponding to the number.

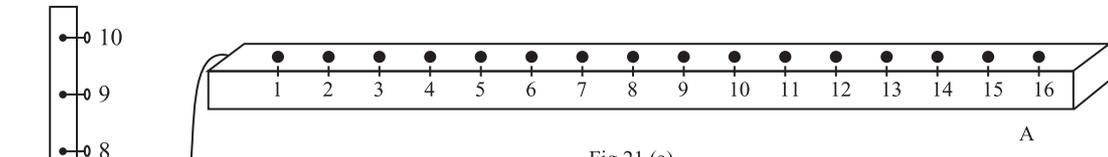


Fig 21 (a)



Fig 21 (b)

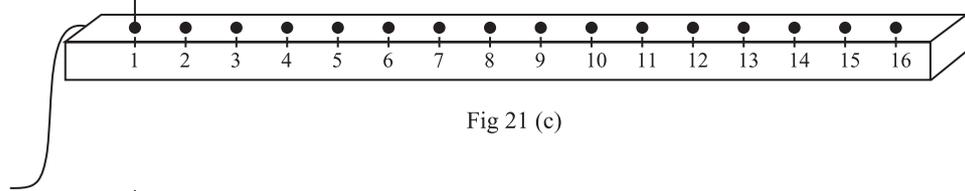


Fig 21 (c)

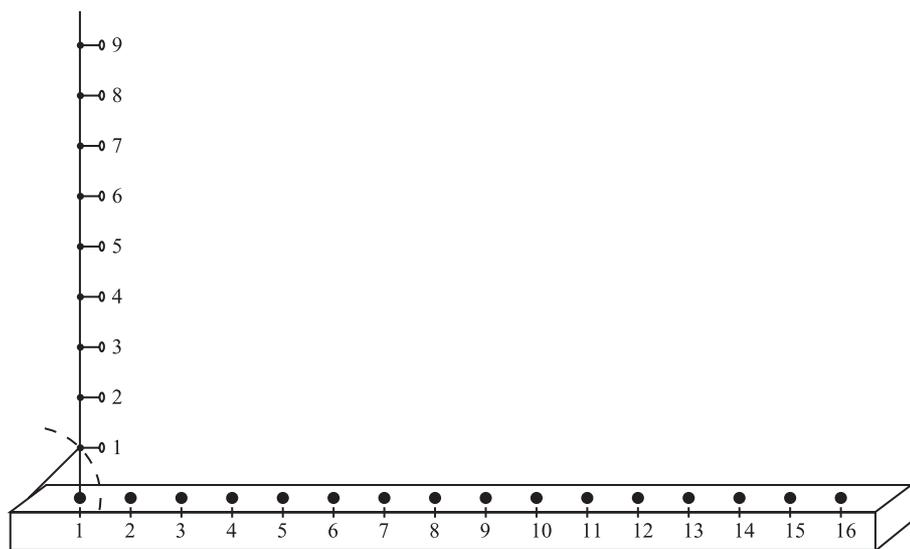
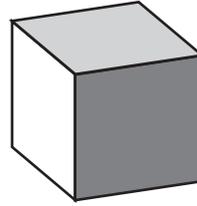


Fig 21 (d)

## Activity 22



# Algebraic Identity

### Objective

To verify the identity  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ , for simple cases using a set of unit cubes.

### Pre-requisite knowledge

1. Express the volume of an object as the number of unit cubes in it.
2. Knowledge of the identity  
 $(a - b)(a^2 + ab + b^2) = (a - b)a^2 + (a - b)(ab) + (a - b)b^2$

### Material Required

27 unit cubes made of wood (dimension is 1 unit  $\times$  1 unit  $\times$  1 unit).

### Procedure

For representing  $a^3 - b^3$

1. Take  $a = 3$  and make a cube of dimension  $3 \times 3 \times 3$  using 27 unit cubes.  
[Fig 22 (a)]
2. Take  $b = 1$  and remove a cube of dimension  $1 \times 1 \times 1$  from  $a^3$ .  
[Fig 22 (b) and Fig 22 (c)]

For representing  $(a - b)(a^2 + ab + b^2)$

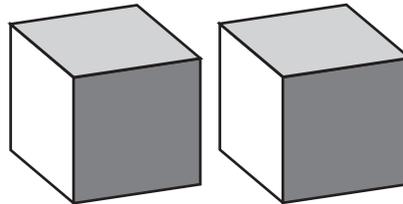
1. We will use the identity  $(a - b)(a^2 + ab + b^2) = (a - b)a^2 + (a - b)(ab) + (a - b)b^2$ . For the given values of  $a$  and  $b$ , the dimensions of cuboids formed in right hand side of the identity are as follows  
 $(a - b)a^2 = 2 \times 3 \times 3$  cubic units  
 $(a - b)(ab) = 2 \times 3 \times 1$  cubic units  
 $(a - b)b^2 = 2 \times 1 \times 1$  cubic units
2. Take the shape obtained by removing  $b^3$  from  $a^3$  and show that it splits into three cuboids of dimensions obtained in the above step. [Fig 22 (d)]

### Observations

1. No. of unit cubes in  $a^3 = 27$
2. No. of unit cubes in  $b^3 = 1$
3. No. of unit cubes in  $a^3 - b^3 = 26$
4. No. of unit cubes in  $(a - b)a^2 = 18$
5. No. of unit cubes in  $(a - b)(ab) = 6$
6. No. of unit cubes in  $(a - b)b^2 = 2$
7. No. of unit cubes in  $(a - b)a^2 + (a - b)(ab) + (a - b)b^2 = 18 + 6 + 2 = 26$



## Activity 23



### Algebraic identity (case I)

#### Objective

To verify the identity  $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ , for simple cases using a set of unit cubes.

#### Pre-requisite knowledge

1. Express the volume of an object as the number of unit cubes in it.
2. Knowledge of the identity  $(a + b)(a^2 - ab + b^2) = a^2(a + b) - ab(a + b) + b^2(a + b)$ .

#### Material Required

36 unit cubes made of wood (dimension is 1 unit  $\times$  1 unit  $\times$  1 unit).

#### Procedure

For representing  $a^2(a + b)$

1. Take  $a = 3$  and  $b = 1$ . Make a cube of dimension  $a^2(a + b)$  i.e.  $3 \times 3 \times 4$  using unit cubes as shown in Fig 23 (a).

For representing  $a^3 + b^3$  as difference between  $a^2(a + b)$  and  $ab(a + b) + b^2(a + b)$

1. Remove a cuboid of dimension  $ab(a + b)$  i.e.  $3 \times 1 \times 4$  [Fig 23 (b)] from Fig 23 (a) as shown in Fig 23 (c).
2. Add a cuboid of dimensions  $b^2(a + b)$  i.e.  $1 \times 1 \times 4$  [Fig 23 (d)] in Fig 23 (c) as shown in Fig 23 (e).
3. Number of cubes remaining is 28.
4. These 28 unit cubes can be rearranged as  $27 + 1 = 3^3 + 1^3$  i.e.  $a^3 + b^3$  as shown in Fig 23 (h).

#### Observations

Number of unit cubes in  $a^2(a + b) = 36$

Number of unit cubes in  $ab(a + b) = 12$

Number of unit cubes in  $b^2(a + b) = 4$

Remaining cubes =  $36 - 12 + 4$

$$= 28$$

$$= 27 + 1$$

$$= 3^3 + 1^3$$

Students verify that

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

#### Learning Outcomes

1. The students obtain the skill of making cuboids using unit cubes.
2. The students will obtain the skill of adding and subtracting the volume of cuboids.

3. Showing the volume of a cube as the sum of cuboids helps them to get a geometric feeling of volume.

**Remark**

1. Teacher can take any value of a and b and verify the result.
2. This activity can be done by taking the formula  $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$  also.
3. The dimensions of cuboid added and removed should be calculated by students..

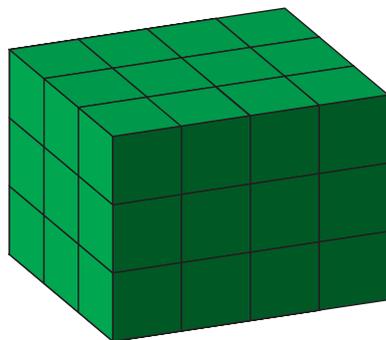


Fig 23 (a)

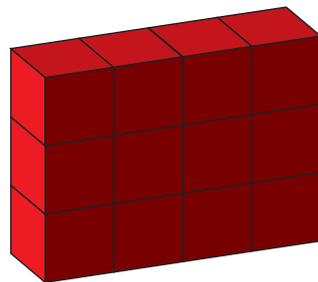


Fig 23 (b)

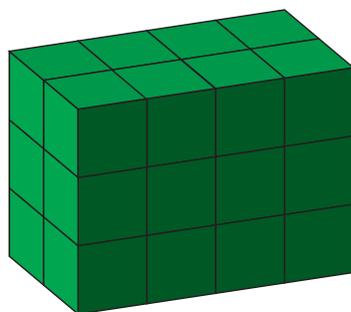


Fig 23 (c)

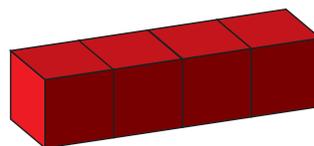


Fig 23 (d)

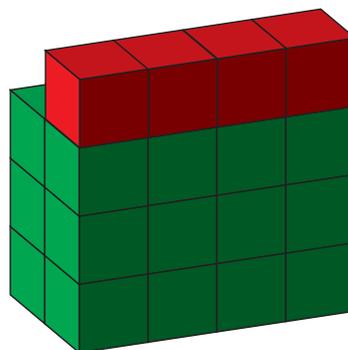


Fig 23 (e)

# Algebraic identity (case II)

## Objective

To verify the identity  $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$  using cuboids and unit cubes.

## Pre-requisite knowledge

Express the volume of an object as the number of unit cubes in it.

## Material Required

64 unit cubes made of wood (dimension is 1 unit  $\times$  1 unit  $\times$  1 unit).

## Procedure

For representing  $(a + b)^3$

1. Take  $a = 3$  and  $b = 1$ . Make a cube of dimension  $4 \times 4 \times 4$  using 64 unit cubes as shown in Fig 23 (f).

For representing  $(a + b)^3 - 3ab(a + b)$

1. Remove a cuboid of dimensions  $ab(a + b)$  i.e.  $3 \times 1 \times 4$  [Fig 23 (g)] three times from Fig 23 (f) as shown in Fig 23 (h).
2. Number of remaining cubes are  $64 - 3 \times (3 \times 1 \times 4) = 64 - 36 = 28$ .
3. These 28 unit cubes can be arranged as  $27 + 1 = 3^3 + 1^3$  i.e.  $a^3 + b^3$  as shown in Fig 23 (h).

## Observations

1. Number of unit cubes in  $(a + b)^3 = 64$
2. Number of unit cubes in  $3ab(a + b) = 3 \times 4 \times 3 = 36$
3. Number of cubes remaining =  $64 - 36 = 28$
4. Number of cubes represented =  $3^3 + 1^3$
5. It is verified that  $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

## Learning Outcomes

1. The students obtain the skill of making cuboids using unit cubes.
2. The students obtain the skill of adding and subtracting the volume of cuboids.
3. Showing the volume of a cube as the sum of cuboids helps them to get a geometric feeling of volume.

## Remark

1. Teachers can take any value of  $a$  and  $b$  and verify the result.
2. Students should find the volume of cuboid by measuring the length, breadth and height.

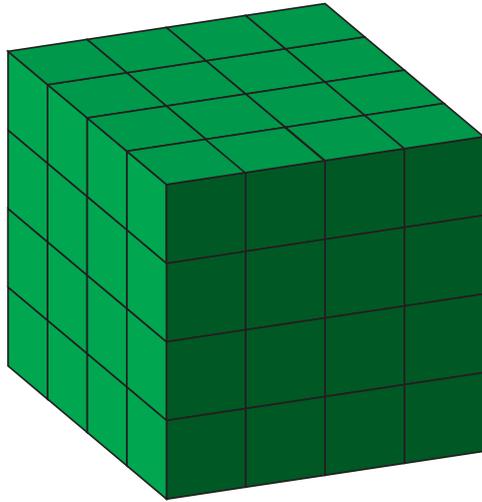


Fig 23 (f)

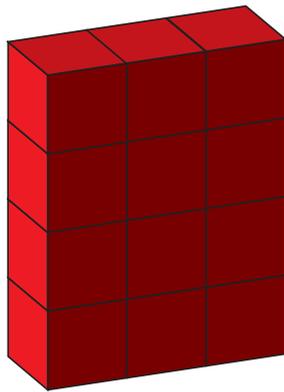


Fig 23 (g)

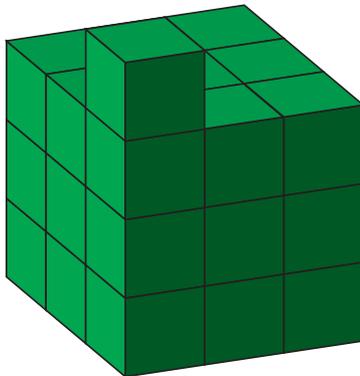
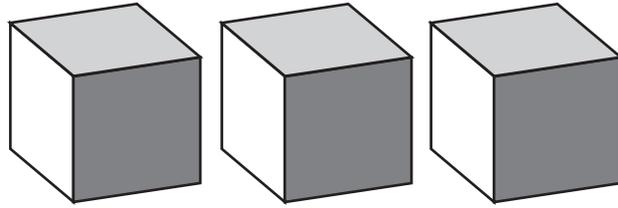


Fig 23 (h)

## Activity 24



# Algebraic identity

### Objective

To verify the identity  $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$  using cuboids and unit cubes.

### Pre-requisite knowledge

1. Express the volume of an object as the number of unit cubes in it.
2. Knowledge of the identity  $(a + b)^3 = a^3 + a^2b + a^2b + a^2b + ab^2 + ab^2 + ab^2 + b^3$

### Material Required

64 unit cubes made of wood (dimension is 1 unit  $\times$  1 unit  $\times$  1 unit).

### Procedure

1. Take  $a = 3$  and make a cube of dimensions  $a^3$  i.e.  $3 \times 3 \times 3$  using 27 unit cubes as shown in Fig 24 (a).
2. Take  $b = 1$  and make a cuboid of dimensions  $a^2b$  i.e.  $3 \times 3 \times 1$ . [Fig 24 (b)] Add this three times in Fig 24 (a) as shown in Fig 24 (c).
3. Make a cuboid of dimensions  $ab^2$  i.e.  $3 \times 1 \times 1$ . [Fig 24 (d)] Add this cuboid three times in Fig 24 (c) as shown in Fig 24 (e).
4. Make a cuboid of dimensions  $b^3$  i.e.  $1 \times 1 \times 1$ . [Fig 24 (f)] Add this cube in Fig 24 (e) as shown in Fig 24 (g).
5. The total number of cubes will be  $64 = 4^3$  i.e.  $(a + b)^3$  as shown in Fig 24 (g).

### Observations

1. Number of unit cube in  $a^3 = 3^3 = 27$
2. Number of unit cube in  $a^2b = 9$
3. Number of unit cube in  $a^2b = 9$
4. Number of unit cube in  $a^2b = 9$
5. Number of unit cube in  $ab^2 = 3$
6. Number of unit cube in  $ab^2 = 3$
7. Number of unit cube in  $ab^2 = 3$
8. Number of unit cube in  $b^3 = 1$
9. Total cubes = 64
10.  $64 = 4^3$
11. It is verified that

$$\begin{aligned}(a + b)^3 &= a^3 + a^2b + a^2b + a^2b + ab^2 + ab^2 + ab^2 + b^3 \\ &= a^3 + 3a^2b + 3ab^2 + b^3\end{aligned}$$

### Learning Outcomes

1. The students obtain the skill of making cuboids using unit cubes.
2. The students obtain the skill of adding and subtracting the volume of cuboids.
3. Showing the volume of a cube as the sum of cuboids helps them to get a geometric feeling of volume.

### Remark

1. Teachers can take any value of a and b and verify the result.
2. The dimensions of cuboid added and removed should be calculated by students.

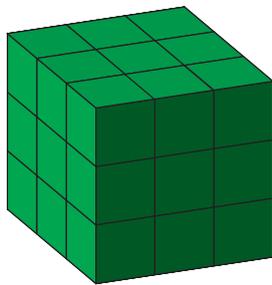


Fig 24 (a)

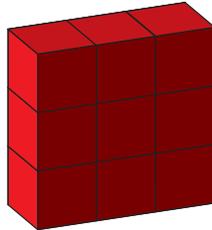


Fig 24 (b)

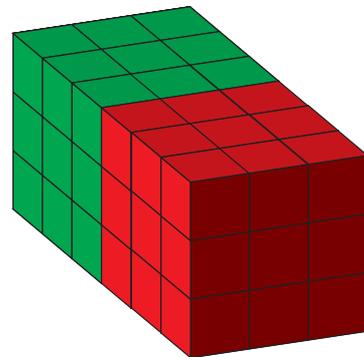


Fig 24 (c)



Fig 24 (d)

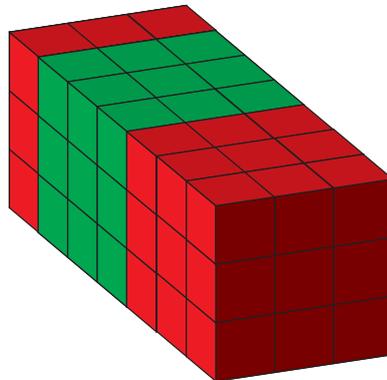


Fig 24 (e)



Fig 24 (f)

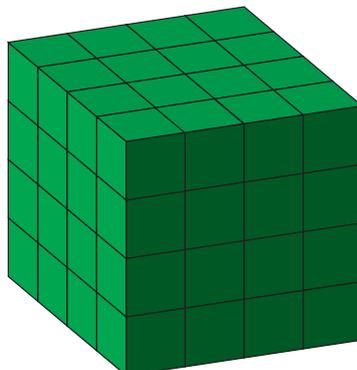
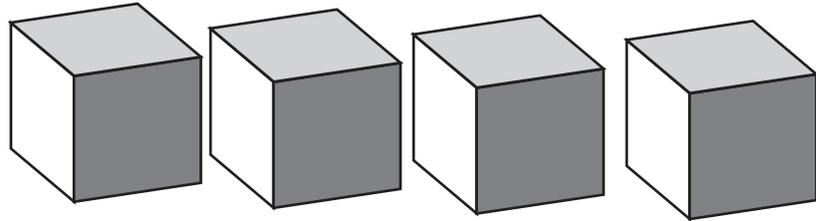


Fig 24 (g)

## Activity 25



# Algebraic identity

### Objective

To verify the identity  $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ , for simple cases using a set of unit cubes.

### Pre-requisite knowledge

1. Express the volume of an object as the number of unit cubes in it.
2. Knowledge of the identity  $a^3 - a^2b - a^2b - a^2b + ab^2 + ab^2 + ab^2 - b^3 = (a - b)^3$ .

### Material Required

64 unit cubes made of wood (dimension is 1 unit  $\times$  1 unit  $\times$  1 unit).

### Procedure

For representing  $a^3$

1. Take  $a = 4$  and  $b = 1$ . Make a cube of dimensions  $4 \times 4 \times 4$  using 64 unit cubes as shown in Fig 25 (a).
2. Remove a cuboid of dimensions  $a^2b$  i.e.  $4 \times 4 \times 1$  [Fig 25 (b)] three times from Fig 25 (a) as shown in Fig 25 (c).
3. Add a cuboid of dimensions  $ab^2$  i.e.  $4 \times 1 \times 1$  [Fig 25 (d)] three times to Fig 25 (c) as shown in Fig 25 (e).
4. Remove a cube  $b^3$  of dimensions  $1 \times 1 \times 1$  [Fig 25 (f)] from Fig 25 (e) as shown in Fig 25 (g).
5. The total number of remaining cubes will be  $27 = 3^3$  i.e.  $a^3$  as shown in Fig 25 (g).

### Observations

1. Number of unit cubes in  $a^3 = 4^3 = 64$
2. Number of unit cubes in cuboid  $a^2b = 4 \times 4 \times 1 = 16$  is removed  
Number of cubes left =  $64 - 16 = 48$
3. Number of unit cubes in cuboid  $ab^2 = 4 \times 1 \times 1$  is added  
Number of cubes left =  $48 + 4 = 52$
4. Number of unit cubes in cuboid  $a^2b = 4 \times 4 \times 1 = 16$  is removed  
Number of cubes left =  $52 - 16 = 36$
5. Number of unit cubes in cuboid  $ab^2 = 4 \times 1 \times 1 = 4$  is added  
Number of cubes left =  $36 + 4 = 40$
6. Number of unit cubes in cuboid  $a^2b = 4 \times 4 \times 1 = 16$  is removed  
Number of cubes left =  $40 - 16 = 24$

7. Number of unit cubes in cuboid  $ab^2 = 4 \times 1 \times 1$  is added

Number of cubes left =  $24 + 4 = 28$

8. Number of unit cube  $b^3 = 1 \times 1 \times 1$  is removed

Number of cubes left =  $28 - 1 = 27$

9.  $27 = 3^3$

10. It is verified that

$$a^3 - a^2b + ab^2 - a^2b + ab^2 - a^2b + ab^2 - b^3 = (a - b)^3$$

$$a^3 - 3a^2b + 3ab^2 - b^3 = (a - b)^3$$

### Learning Outcomes

1. The students obtain the skill of making cuboids using unit cubes.
2. The students obtain the skill of adding and subtracting the volume of cuboids.
3. Showing the volume of a cube as the sum of cuboids helps them to get a geometric feeling of volume.

### Remark

Teachers can take any value of  $a$  and  $b$  and verify the result.

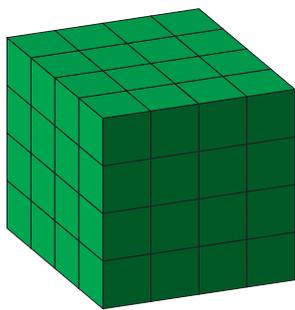


Fig 25 (a)

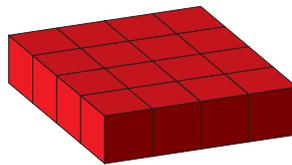


Fig 25 (b)

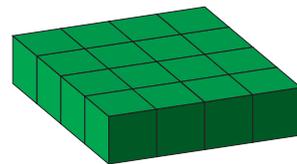


Fig 25 (c)

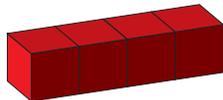


Fig 25 (d)

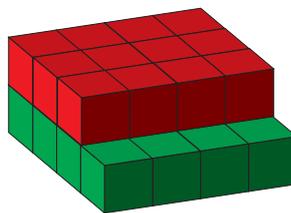


Fig 25 (e)



Fig 25 (f)

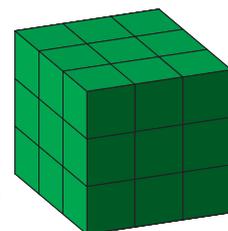
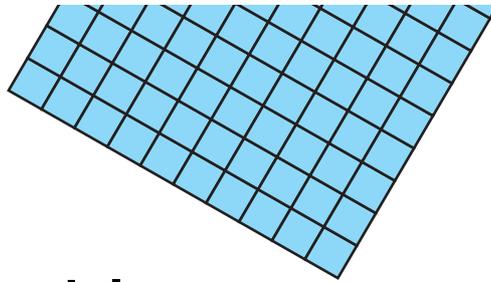


Fig 25 (g)

## Activity 26



# Factorisation of polynomials

### Objective

To interpret geometrically the factors of a quadratic expression of the type  $x^2 + bx + c$  using square grids, strips and paper slips.

### Pre-requisite knowledge

1. Splitting the middle term of a quadratic polynomial.
2. Area of a rectangle

### Material Required

Square grids, strips and paper slips.

### Procedure

#### Case I

Take  $b = 5$ ,  $c = 6$

Polynomial is  $x^2 + 5x + 6$

Now find two numbers whose sum is 5 and product is 6 i.e. 3 and 2.

therefore,  $x^2 + 5x + 6 = x^2 + 3x + 2x + 6$

1. Take a square grid of dimension  $(10 \times 10)$ . It represents  $x^2$  as shown in Fig 26 (a).
2. Add 3 strips of each dimensions  $x \times 1$  as shown in Fig 26 (b).
3. The area of rectangle formed in Fig 26 (b) represents  $x^2 + 3x$ .
4. Add 2 strips of dimensions  $2 \times x$  as shown in Fig 26 (c). Now the total area =  $x^2 + 3x + 2x$ .
5. Add 6 slips of dimensions  $(1 \times 1)$  i.e.  $6 \times 1$  to complete the rectangle as shown in Fig 26 (d).

### Observations

The students will observe that

Area of new rectangle =  $x^2 + 3x + 2x + 6$

$(x + 3)(x + 2) = x^2 + 3x + 2x + 6$

therefore,  $x^2 + 5x + 6 = (x + 3)(x + 2)$

#### Case II

Take  $b = 1$ ,  $c = -6$

Polynomial is  $x^2 + x - 6 = x^2 + 3x - 2x - 6$

1. Take a square grid of dimension  $(10 \times 10)$ . It represents  $x^2$  as shown in Fig 26 (e).
2. Add 3 strips of dimensions  $1 \times x$  as shown in Fig 26 (f).
3. The area of rectangle formed in Fig 26 (f) represents  $x^2 + 3x$ .

4. Shade 2 strips of dimensions  $1 \times x$  as shown in Fig 26 (g).
5. Remove 6 slips of dimensions  $1 \times 1$  so as to complete the rectangle. We have new rectangle of dimensions  $(x - 2) \times (x + 3)$  as shown in Fig 26 (h).

### Observations

The students will observe that

$$\text{Area of new rectangle} = x^2 + 3x - 2x - 6 = (x + 3)(x - 2)$$

$$\text{therefore, } x^2 + x - 6 = (x + 3)(x - 2)$$

### Learning Outcomes

The students learn the geometrical meaning of the process of factorization of a quadratic expression. The three terms in the polynomial  $x^2 + bx + c$  correspond to a square and two rectangles. The polynomial is factorisable if the three figures can be arranged to form a single rectangle whose sides are the factors of the given polynomial.

### Remark

1. Teacher may choose polynomial of the type  $x^2 + bx + c$  taking other suitable values of  $b$  and  $c$  for the activity (where  $b$  or  $c$  is negative).
2. 10 slips = 1 strip, 10 strips = 1 grid.

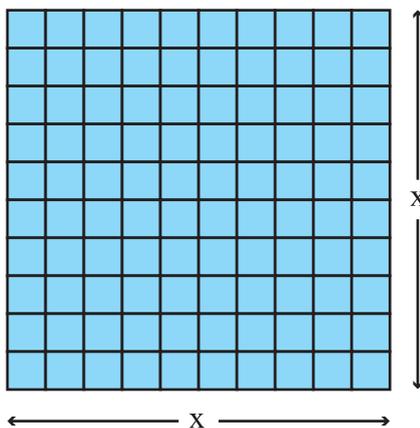


Fig 26 (a)

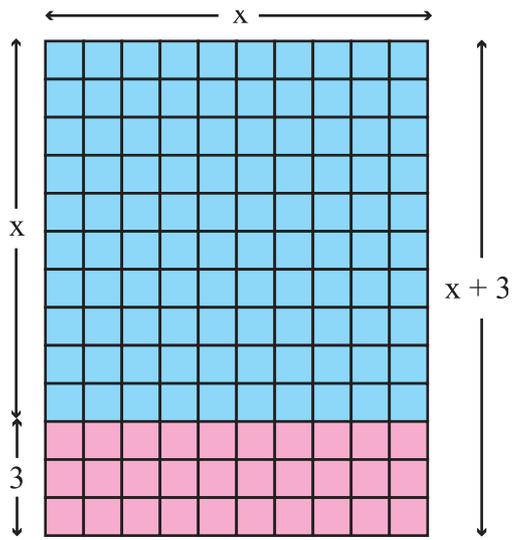


Fig 26 (b)

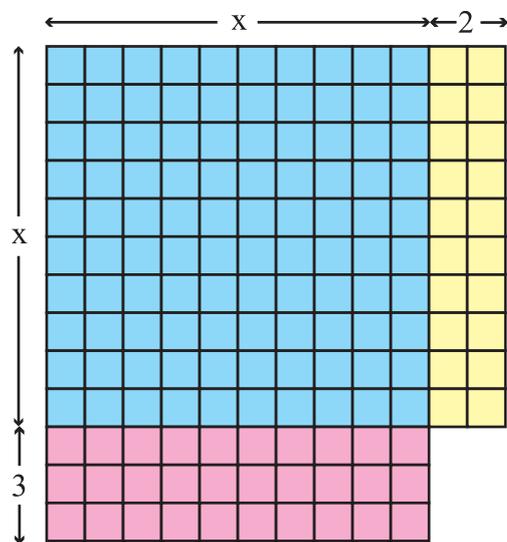


Fig 26 (c)

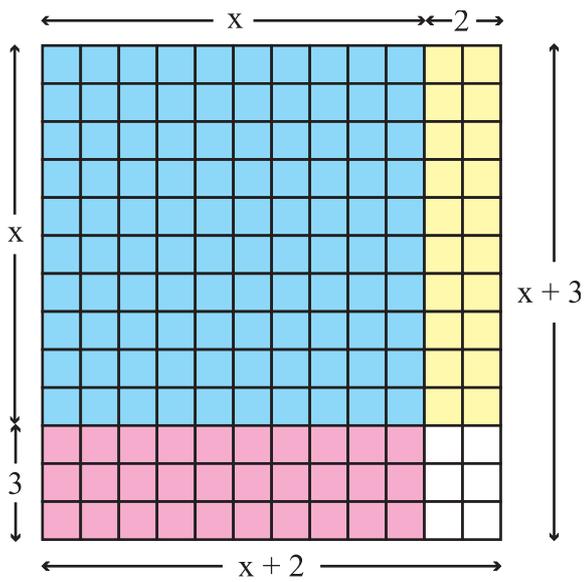


Fig 26 (d)

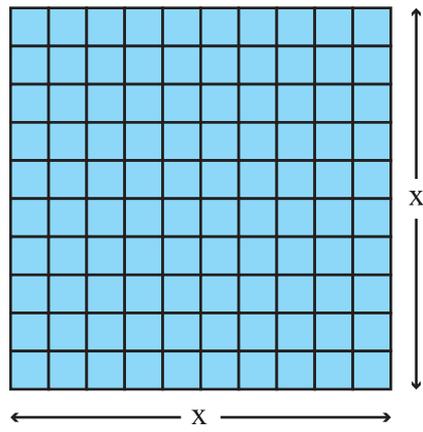


Fig 26 (e)

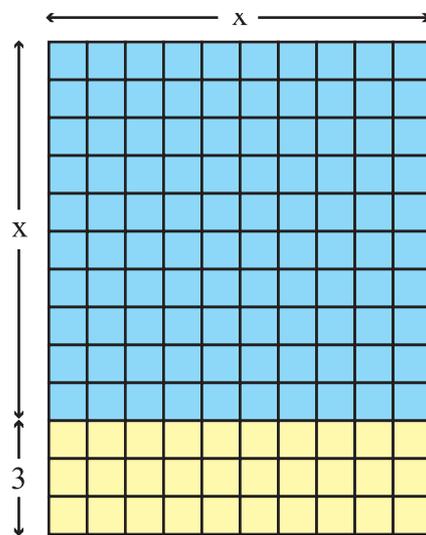


Fig 26 (f)

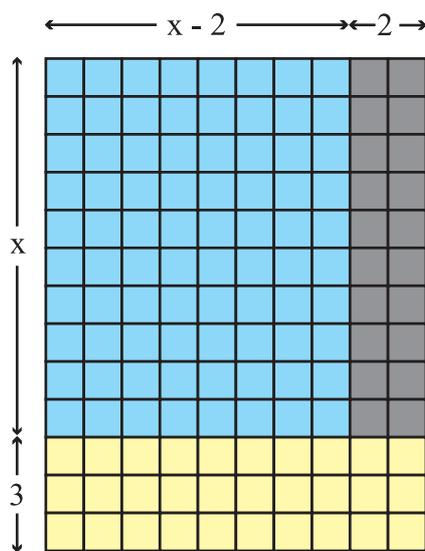


Fig 26 (g)

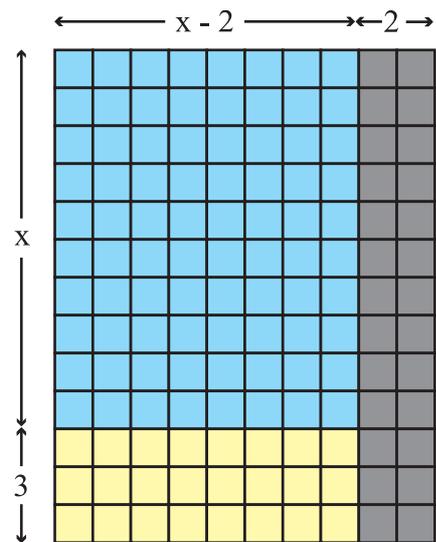
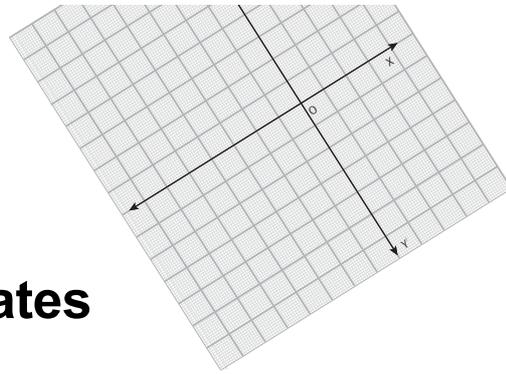


Fig 26 (h)

## Activity 27



# Quadrants and co-ordinates

### Objective

To obtain the mirror image of a given geometrical figure with respect to the x-axis and the y-axis.

### Pre-requisite knowledge

Plotting of points on the graph paper, idea of mirror image points about a given line.

### Material Required

Graph paper, pencil, ruler.

### Procedure

1. Write the co-ordinates of the labeled points (A, B, C, D...) in the given Fig 27 (a).
2. Plot the mirror images of the points (A, B, C, D...) with respect to the x-axis and label the co-ordinates obtained.
3. Join the co-ordinates obtained in step 2 in order to get the mirror image of the given figure with respect to the x-axis.
4. Repeat the process to get the mirror image of the given figure with respect to the y-axis.

### Observations

The students will observe the following

1. When the mirror image of a figure is obtained with respect to the y-axis, the y-coordinate remains the same.
2. When the mirror image of a figure is obtained with respect to the x-axis, the x-coordinate remains the same.

### Learning Outcomes

1. The students will get practice of plotting points with given co-ordinates.
2. The students will develop a geometrical intuition for reflection symmetry.
3. The students will get an idea of developing symmetrical designs.

### Remark

1. The teacher may suggest any figure other than Fig 27 (a) for doing this activity in the mathematics laboratory.
2. The teacher may ask the children to find the mirror image of a figure with respect to any other line e.g.  $x = y$ . (Here we have taken the x and y axis to get the reflections of points).

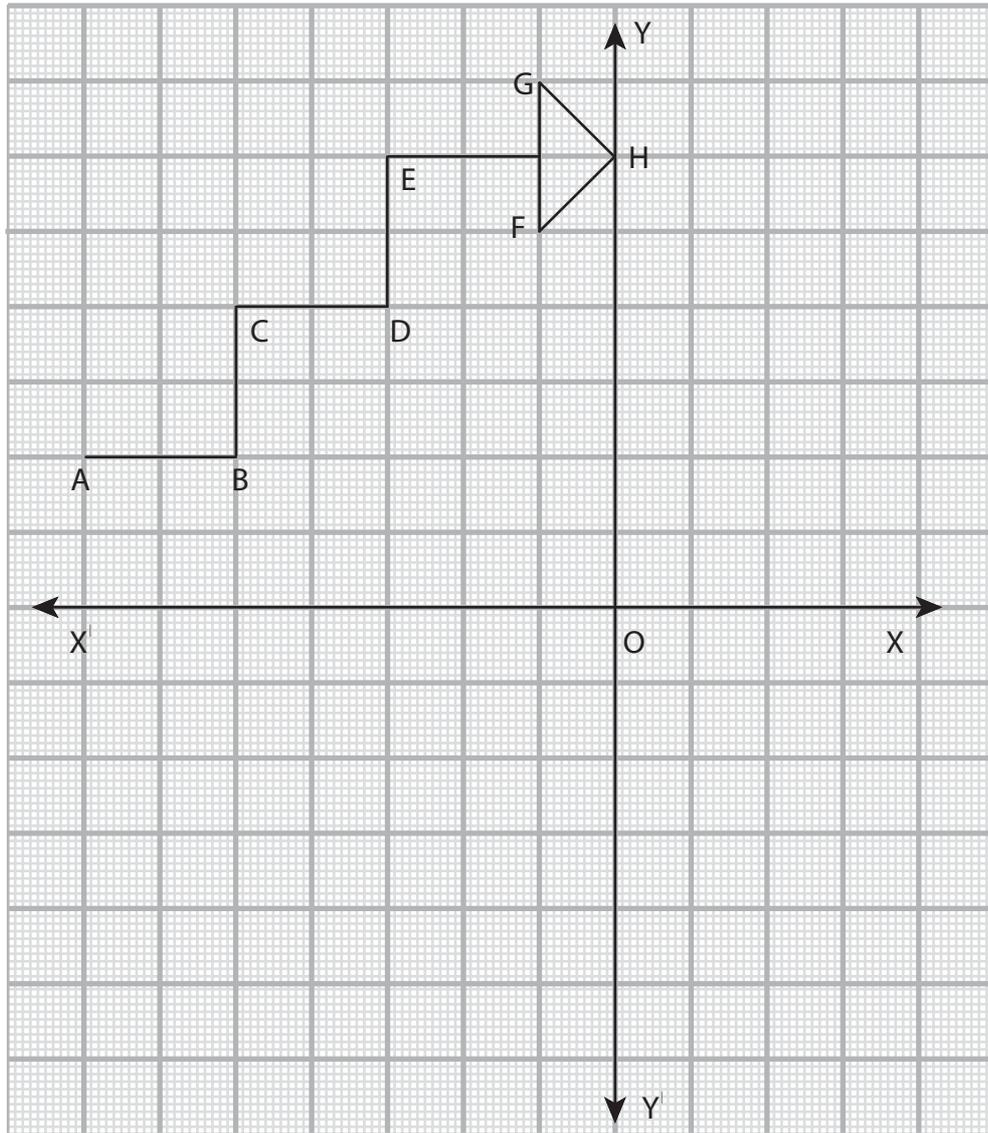


Fig 27 (a)

## Group Activity I

# Graph and Percentage

### Objective

To find the percentage of the students in a group of students who write faster with their left / right hand.

### Pre-requisite knowledge

1. Knowledge of plotting the points on graph paper.
2. Knowledge of calculating percentage.

### Procedure

1. Ask the students to take a paper and a pen.
2. Ask each student to write a letter (say 'a') or a digit (say '2') for 25 seconds with his/her right hand. Ask them to count the total number of digits/letters written by them.
3. Repeat the same experiment with left hand for same duration.
4. Each student will record this data in a table shown below.

No.	Name of the student	No. of digits written by student with right hand	No. of digits written by student with left hand
1			
2			
3			

5. Take the number of digits written by their right hand as 'x' and the number of digits written by their left hand as 'y'.
6. Plot the co-ordinate (x, y) for every student on the graph paper.
7. Draw the line  $x = y$  on the graph paper.
8. From the graph count the number of points which are below the line  $x = y$  and the number of points which are above the line  $x = y$ .

### Observations

The students will observe that

1. When the value of x is greater than y it means that student writes faster with his/her right hand.
2. When the value of y is greater than x it means that student writes faster with his/her left hand.
3. They will also determine percentages for each group.

### **Learning outcomes**

1. Through this activity students are gaining the experience of collecting data, calculating percentage and plotting graph within a realistic content.
2. They will also apply simple mathematical ideas to a practical situation.

### **Remark**

Teachers can give any other realistic situation which can be graphically analysed.

## **Group Activity 2**

### **Measure-up**

#### **Objective**

To help the students establish interesting mathematical relationships by measuring some parts of the body.

#### **Background**

In a class, teacher wants to know the personal mathematics (measurement of some parts of the body) of her students. She makes a group of two students. The class consists of 40 students. So, randomly 20 groups are made. Every member of the group has to perform the activity.

Using the situation given above, some queries can be asked as follows -

1. What is the average height of the students in class IX ?
2. What is the average weight of these students ?
3. What percent of squared students is there in the class ?
4. What is the average shoe size of these students ?
5. What is the average neck size of these students ?
6. What is the average wrist size of the students ?

#### **Procedure**

Measurement is taken by both the members of the students in the group using measuring tapes in the following format

Height =

Out stretched arm length =

Ratio = height / our stretched arm length.

Foot =

Palm =

Ratio =

Wrist =  
 Neck =  
 Ratio =

Elbow to finger =  
 Head span =  
 Ratio =

Weight is taken  
 Weight =  
 Height =  
 Ratio =

Weight of the students is only taken on school weighing machine.

**Observations**

A

Number of squared students in the class \_\_\_\_\_  
 Number of rectangled students in the class \_\_\_\_\_

B

Average height = \_\_\_\_\_  
 Average weight = \_\_\_\_\_  
 Average foot size = \_\_\_\_\_  
 Average neck size = \_\_\_\_\_  
 Average wrist size = \_\_\_\_\_

C

Students will make a chart for their class as follows

Height		Foot Size		Weight		Neck Size		Wrist Size	
Below average	Above average								
%	%	%	%	%	%	%	%	%	%

D

Students will plot the graph of the following

- 1) height vs weight
- 2) foot vs palm
- 3) neck vs wrist

E

Students will find the average ratios of the following

- 1) Height : Weight
- 2) Height : Out stretched arm length
- 3) Foot : Palm

Every student will write his/her observation of the data they have collected and analysed.

They should also conclude some results from the graphs they have obtained.

### **Learning Outcomes**

1. The students will gain the experience of obtaining the data of their personal physical structure.
2. This would encourage them to see what are standard measurements and how much deviated they are from it.
3. They will also apply all the mathematical concepts that they are learning in their school statistics.
4. This helps them to learn mathematics in a realistic way. (how much they are closer to the standard measurement)

### **Remark**

Squared person → Squared person is a person whose measurement of stretched arm length and height is same.

Rectangled person → Rectangled person is a person whose measurement of stretched arm length and height is not same.

## Project 1

### Observing interesting patterns in cricket match

#### Objective

Comparison of the performance of two teams in a one-day international cricket match.

#### Project Details

Data of scores can be collected to study various aspects such as

1. Performance of both the teams according to run rates per over, wicket rates per over.
2. Investigate if run rates are uniform for both the teams.
3. Investigate the run rates for various bowling techniques used by bowlers. (Fast, slow, spin bowlers)

#### Methods

1. All the details and the data can be collected while watching the game, listening the commentary and while reading the reports in the newspaper about the match.
2. Collected data can be tabulated in the form of grouped data, represented by the histogram, bar graphs, frequency polygon, pie chart etc.

#### Result

Inferences can be drawn from the above presentation of data about the batting pattern, bowling pattern, etc.

#### Acknowledgements

T.V. channel / Radio channel, commentators, guide.

#### Reference

Students should describe all the sources he/she used to collect and compile the data.

## Project 2

### Design a crossword puzzle with mathematical terms

#### Objective

To review mathematics vocabulary, to give the opportunity for creative expression in designing puzzles, to act as a means of motivating the study of a given unit and to give recreation.

### Description

Take a square grid (9 × 9) where a few words are connected horizontally and vertically. First compile a list of the terms. Then decide on the dimensions for the finished puzzle, preferably on squared paper with blocks measuring at least a half-inch on each side. A design may or may not be blocked out before inserting the terms. The words showed be connected but may stand alone if they fit into the pre-determined spaces allowed for the puzzle. [Fig P2 (a)]

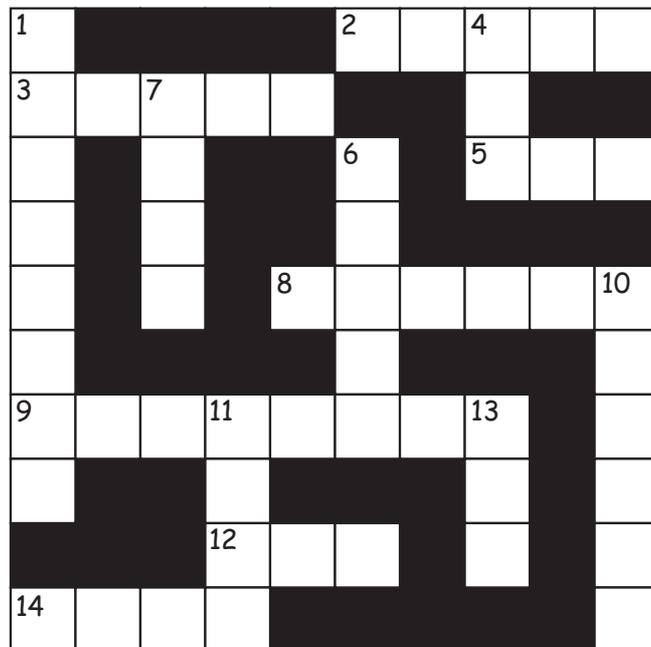


Fig P2 (a)

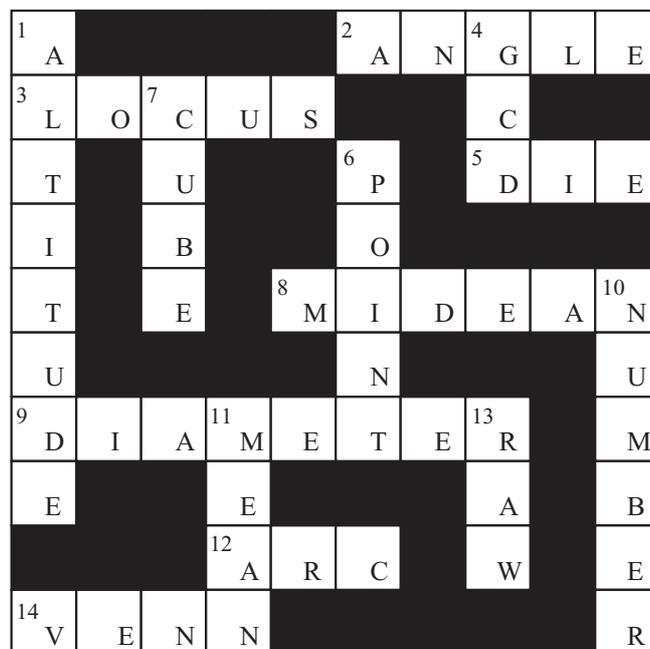


Fig P2 (b)

The puzzle, which is given [Fig P2 (b)] may be used as guideline for framing of puzzles based on

1. Mathematical operations.
2. Terms based on geometrical shapes.

### **ACROSS**

2. Another word for inclination
3. The way a path moves
5. A cube numbered 1, 2, 3, 4, 5, and 6 on the faces
8. Average finding by using statistical data
9. The longest chord of a circle
12. Part of a circle is called
14. In set theory, the name of the diagram is called

### **DOWN**

1. Height of a triangle
4. Abbreviation of greatest common denominator
6. A dot on a piece of paper
7. A solid figure has six faces
10. Counting objects are called
11. Another measure of central tendency
13. Information can be used in statistics as

## **Project 3**

### **A measuring task**

#### **Objective**

To investigate your local athletics track to see whether it is worked fairly for runners who start on different lines.

#### **Pre-requisite knowledge**

1. When athletes run around a 400 m track, the competitors in the outside line start ahead of those in an inside line because they have further to run.
2. Distance between the starting positions is called a stagger.
3. Shape of the track can be considered to be rectangular with a semi circle at each end.

## Method

Student uses a measuring tape (non stretchable).

1. Measure the length of the two straight parts of the track.
2. Measure the distance between two straight parts of the track.
3. Explain how these measurements help in finding out the radius 'r' of each semi circle.
4. Calculate the distances for the two round parts of the track.
5. Distance around the track for runner in the innermost line.
6. Now measure the width 'w' of a line. What will be the radius of the next line in terms of 'r' and 'w'.
7. Calculate the circumferences for the next line.
8. Compare the two distances, the 2 runners in line 1 and line 2. How to make and find the stagger ?
9. Investigate to see whether the stagger is the same for each successive line.

Based on all collected and calculated data students will conclude whether it is worked fairly for runners who start on different lines on the athletics tracks.

## Project 4

### Project in History of Mathematics

The students can choose several topics from history of mathematics, for doing a project. For instance the topic can be about an Indian mathematician or the concept of zero in various ancient civilizations. In what follows we give two illustrative examples.

#### Example 1

### Pythagoras theorem

#### Objective

Study of various aspects of Pythagoras theorem

#### Description

Study some or all the following aspects of the theorem:

1. Biography of Pythagoras.
2. Statement of the theorem.
3. Proofs of the theorem that can be given by cutting and pasting paper/ paper folding.
4. Everyday illustrations/ applications of the theorem.
5. Pythagorean triplets of integers.

Methodology is primarily literature survey/ library work, besides using paper folding techniques.

### **Results and findings**

The student organises the information. They gather it in a systematic way and group it under different chapters of a project report.

### **Acknowledgements**

The students should mention honestly the names of individuals who have helped them.

### **References**

Students should describe all the sources they used to collect and compile the data.

## **Example 2**

# **History of the number $\pi$**

### **Objective**

Investigation of various historical aspects of the number  $\pi$ .

### **Description**

1. Knowledge about  $\pi$  in various ancient civilizations.
2. Approximations for  $\pi$ .
3. Circle and  $\pi$ .
4. Famous mathematical problems featuring  $\pi$ .

Methodology is primarily study of material on the history of mathematics.

Results and findings are organized under various chapters resulting in a project report.

Acknowledgements are listed by the student to thank the individuals/ institutions for the help he/she received.

### **References**

Students should describe all the sources they used to collect and compile the data.

